Workshops:
The heart of the MagiKats Programme

Maths Stage 5

Every student is assigned to a Stage, based on their academic year and assessed study level.

Stage 5 students have completed all Stage 4 materials.

The sheets in this pack are a small sample of what is available! These are only samples of the student's worksheets - our teaching methods include discussion and hands-on activities.
Core skills sheets are also provided for independent completion by each student (usually at home).

At this level, students study all the more advanced topics needed to take them to about age 16. Stage 5 material follows on from that in Stage 4 and is offered from age 14. It is only completed by those expecting to continue studying mathematics to a more advanced level.

Student: Date:

Maths Stage 5: Factorisation

Sheet 1

Factorising Quadratic Equations

Warm up questions: expand and simplify.

1) $(x + 2)(x + 4)$

2) $(a + 6)(a - 7)$

3) $(s - 6)(s + 11)$

4) $(5y - 3)(y - 5)$

5) $(7t + 6)(5t + 8)$

6) $(x - 3)(3x - 2)$

7) $(2a + 3)(a - 8)$

8) $(2x + 1)(x + 5)$

9) $(3x - 2)(x - 4)$

10) $(x - 5)(4x + 2)$

Any expression of the form $ax + b$, where a and b are numbers, is called a **linear expression in x**.

Look at your answers and you will see that when two linear expressions in x are multiplied, the result usually contains three terms: a term in x^2, a term in x and a number.

Expressions of this form, i.e. $ax^2 + bx + c$ where a, b and c are numbers and $a \neq 0$, are called **quadratic expressions in x**.

Knowing that the product of two linear brackets is quadratic, we should be able to work backwards to factorise a quadratic. For instance, given a quadratic such as $x^2 - 5x + 6$, we could try to find two linear expressions in x whose product is $x^2 - 5x + 6$.

To be able to do this we need to understand the relationship between what is inside the brackets and the resulting quadratic.

Student: Date:

Maths Stage 5: Factorisation

Sheet 2

Looking at the last three questions in your warm up:

$$(2x + 1)(x + 5) = 2x^2 + 11x + 5 \qquad (1)$$

$$(3x - 2)(x - 4) = 3x^2 - 14x + 8 \qquad (2)$$

$$(x - 5)(4x + 2) = 4x^2 - 18x - 10 \qquad (3)$$

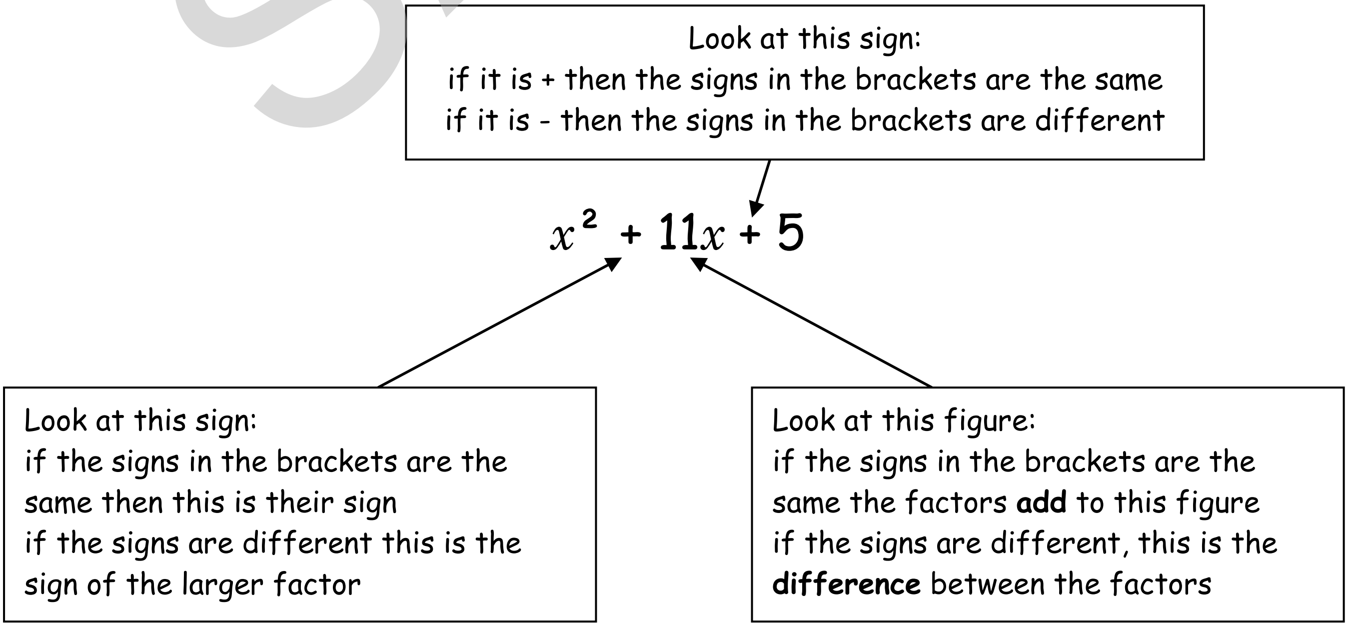
The first thing to notice about the quadratic in each example is that:

- The coefficient of x^2 is the product of the coefficients of x in the two brackets.
- The number term is the product of the numbers in the two brackets.
- The coefficient of x is the sum of the coefficients formed by multiplying the x term in one bracket by the number term in the other bracket.

The next thing to notice is the relationship between the signs.

- + signs throughout the quadratic come from + signs in both brackets, as in (1).
- A + number term and a - coefficient of x in the quadratic come from a - sign in each bracket, as in (2).
- A - number term in the quadratic comes from a - sign in one bracket and a + sign in the other, as in (3).

This ties in neatly with information that you already know: when multiplying two numbers, if their signs are the same then the result is positive but if the signs are different, the result is negative.



Look at this sign:
if it is + then the signs in the brackets are the same
if it is - then the signs in the brackets are different

$$x^2 + 11x + 5$$

Look at this sign:
if the signs in the brackets are the same then this is their sign
if the signs are different this is the sign of the larger factor

Look at this figure:
if the signs in the brackets are the same the factors **add** to this figure
if the signs are different, this is the **difference** between the factors

Student: Date:

Maths Stage 5: Factorisation

Sheet 3

Factorising Quadratic Equations: another way of looking at it!

$(x + a)(x + b) = x^2 + (a+b)x + ab$

Therefore

1) $x^2 + (a+b)x + ab = (x + a)(x + b)$

2) $x^2 - (a+b)x - ab = (x + a)(x - b)$ (where b is the larger digit)

3) $x^2 + (a+b)x - ab = (x - a)(x + b)$ (where b is the larger digit)

4) $x^2 - (a+b)x + ab = (x - a)(x - b)$

Examples

1) $x^2 + 7x + 10 = (x + 2)(x + 5)$

2) $x^2 - 3x - 10 = (x + 2)(x - 5)$

3) $x^2 + 3x - 10 = (x - 2)(x + 5)$

4) $x^2 - 7x + 10 = (x - 2)(x - 5)$

When factorising a quadratic in the form $x^2 + (a+b)x + ab$:

Look at the last term – is it +ve or –ve?

- If it is +ve then the signs in the two brackets are the same – either both + or both -. Their sign is the same as the sign of the x term and the sum of the factors (a+b) is the coefficient of x.

- If it is –ve then the signs are different – one is + and one is -. The larger factor has the same sign as the x term and the difference between the factors (a-b) is the coefficient of x.

Examples: factorising $x^2 + 3x - 10$

last term is –ve so different signs $= (\quad + \quad)(\quad - \quad)$
and factors of 10 (1x10, 2x5) must have a difference of 3
factors for use must be 2x5 and larger one is +ve so $= (x + 5)(x - 2)$

Examples: factorising $x^2 + 11x + 10$

last term is +ve so signs both same as x term $= (\quad + \quad)(\quad + \quad)$
and factors of 10 (1x10, 2x5) must add to 11
factors for use must be 1x10 so $= (x + 1)(x + 10)$

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Maths Stage 5: Factorisation

Sheet 4

Examples:

1) Factorise $x^2 - 5x + 6$

The x term in each bracket is x as x^2 can only be $x \times x$.
The + sign at the end means that the signs are the same in each bracket
The - sign means both signs are -, so $x^2 - 5x + 6 = (x - \ )(x - \ )$
The numbers in the brackets (factors of 6) could be 6 and 1 or 2 and 3.
The signs are the same so add to 5, i.e. the numbers must be 2 and 3.

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Expanding the brackets back checks that they are correct.

2) Factorise $x^2 - 3x - 10$

The x term in each bracket is x
The - sign at the end means that the signs are different $\Rightarrow x^2 - 3x - 10 = (x - \ )(x + \ )$
The numbers could be 10 and 1 or 5 and 2.
The signs are different so we need a difference of 3, i.e. they are 5 and 2.

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

Expanding the brackets back checks that they are correct.

Factorise, and then check your answer.

1) $x^2 + 8x + 15$

2) $x^2 + 7x + 6$

3) $x^2 - 10x + 9$

4) $x^2 + 8x + 12$

5) $x^2 + 5x - 14$

6) $x^2 - 4x - 5$

7) $x^2 + 9x + 14$

8) $x^2 - 9$

9) $x^2 + 4x + 4$
think about how to write this answer!

10) $x^2 - 3x - 18$

11) $x^2 - 16$

12) $2x^2 - 3x + 1$

13) $9x^2 - 6x + 1$

14) $9 + 6x + x^2$

Student: Date:

Maths Stage 5: Factorisation

Sheet 5

Factorise the following equations:

1) $x^2 - 7x + 12$

2) $x^2 - 6x - 16$

3) $x^2 + 3x + 2$

4) $x^2 - x - 2$

5) $x^2 + 9x - 10$

6) $x^2 - 6x - 7$

When you have factorised, you can then go on to solve the equation (find the value for x).

Look at $x^2 - 3x - 10 = (x - 5)(x + 2)$

Suppose the question says

Factorise and solve $x^2 - 3x - 10 = 0$

Once you have factorised, you know that $x^2 - 3x - 10 = (x - 5)(x + 2)$

So $(x - 5)(x + 2) = 0$

If the product of any two numbers is 0 then one or other of the numbers is 0 so

Either $(x - 5) = 0$
So $x = 5$

Or $(x + 2) = 0$
So $x = -2$

Student: Date:

Maths Stage 5: Factorisation

Sheet 6

Try these. Be sure to keep all your workings clear and line up = signs under each other.

Factorise and solve the following equations:

1) $x^2 - 14x + 24 = 0$

2) $x^2 - 5x + 6 = 0$

3) $x^2 + 11x + 18 = 0$

4) $x^2 - 4x + 3 = 0$

5) $2x^2 - 19x + 35 = 0$

6) $3x^2 + 14x + 8 = 0$

Student: Date:

Maths Stage 5: Factorisation

Sheet 7

Extension Work: Harder Factorising

When the number of possible combinations of terms for the brackets increases, common sense considerations can help to reduce the possibilities.

For example, if the coefficient of x in the quadratic is odd, then there must be an even number and an odd number in the brackets.

Example:

Factorise $12 - x - 6x^2$

The x terms in the brackets could be $6x$ and x, or $3x$ and $2x$, one positive and the other negative.

The number terms could be 12 and 1 or 3 and 4 (not 6 and 2 because coefficient of x in the quadratic is odd).

Now we try various combinations until we find the correct one.

$$12 - x - 6x^2 = (3 + 2x)(4 - 3x)$$

Factorise:

1) $6x^2 + x - 12$

2) $4x^2 + 3x - 1$

3) $4x^2 - 12x + 9$

4) $25x^2 - 16$

5) $5x^2 - 61x + 12$

6) $3 + 2x - x^2$

7) $1 - x^2$

8) $x^2 + 2xy + y^2$

9) $4x^2 - 4xy + y^2$

10) $36 + 12x + x^2$

Direct and Inverse Proportion

When working on proportion at a higher level, we usually express it in terms of unknowns. Direct proportion is very straight forward. One variable (e.g. y) can be calculated as the other variable (e.g. x) multiplied by some constant. An example would be $y = 3x$

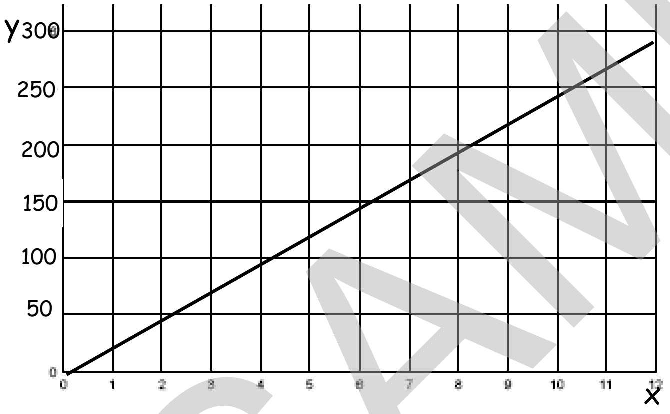
Inverse proportion is a little more complicated. Here, one variable (e.g. y) can be calculated as one over the other variable (e.g. $1/x$) multiplied by some constant.

An example would be $y = 5/x$

Direct Proportion: $y = kx$

Both Increase Together

The graph of y against x is a straight line through the origin: $y = kx$



In both cases k is a constant.

In a table of values the multiplier is the same for x and y, so if you double one of them, you double the other, and so on.

$$\begin{tikzcd}
x & 2 \arrow[r, "\times 3"] & 6 \arrow[rr, bend left, "\times 2"] & 8 & 12 & 14 \\
y & 3 \arrow[r, "\times 3"'] & 9 \arrow[rr, bend right, "\times 2"'] & 12 & 18 & 21
\end{tikzcd}$$

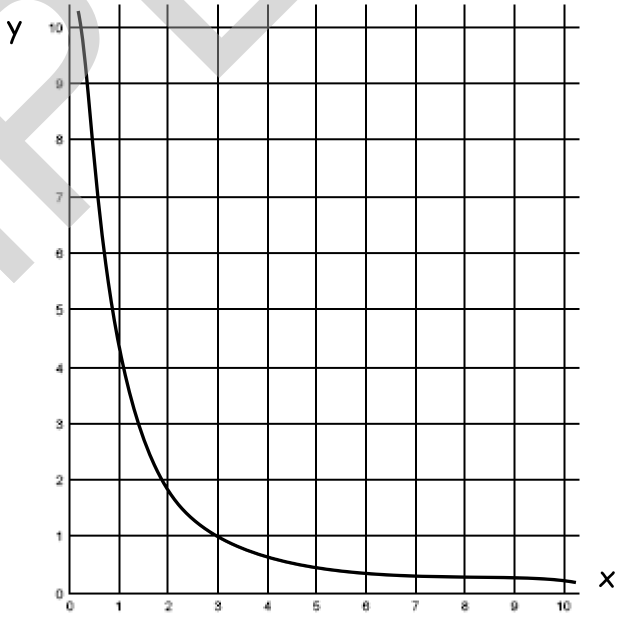
The **RATIO** x/y is the same for all pairs of values.

$$\frac{2}{3} = \frac{6}{9} = \frac{8}{12} = \frac{12}{18} = \frac{14}{21} = 0.6667$$

Inverse Proportion: $y = k/x$

One Increases, One Decreases

Sketch graph of $y = k/x$



In a table of values the multiplier for one of them becomes a divider for the other, so if you double one, you half the other etc.

$$\begin{tikzcd}
x & 2 \arrow[r, "\times 3"] & 6 \arrow[rr, bend left, "\times 2"] & 8 & 12 & 40 \\
y & 30 \arrow[r, "\div 3"'] & 10 \arrow[rr, bend right, "\div 2"'] & 7.5 & 5 & 1.5
\end{tikzcd}$$

The **PRODUCT** xy (x times y) is the same for all pairs of values.

2x30= 6x10= 8x7.5= 12x5= 40x1.5= 60

Variation

Sometimes your questions may be phrased like these:

"y is proportional to the square of x" "t is proportional to the square root of h"

"$\mathcal{D}$ varies with the cube of t" "$\mathcal{V}$ is inversely proportional to r cubed"

There is a method you must remember to successfully deal with these sorts of questions:

Method

1) **Convert the sentence into a proportionality** using the symbol " $\propto$ " which means "is proportional to".

2) **Replace " $\propto$ " with "=k"** to make an **equation**:

The above examples would become:	Proportionality	Equation
y is proportional to the square of x	$y \propto x^2$	$y = kx^2$
t is proportional to the square root of h	$t \propto \sqrt{h}$	$t = k\sqrt{h}$
$\mathcal{D}$ varies with the cube of t	$\mathcal{D} \propto t^3$	$\mathcal{D} = kt^3$
$\mathcal{V}$ is inversely proportional to r cubed	$\mathcal{V} \propto 1/r^3$	$\mathcal{V} = k/r^3$

3) **Find a pair of values of x and y** somewhere in the question, and **substitute them into the equation**.

4) **Calculate the value of k and put it into the equation** and it's now ready to use.
Example: $y = 3x^2$

5) Finally, find either x or y (depending on which value is already supplied in the question).

Example:

The time taken for a cat to crawl through a drain pipe is inversely proportional to the square of the diameter of the pipe. If he took 25 seconds to crawl through a pipe of diameter 0.3m, how long would it take him to get down one of 0.2m diameter?

Solution:

1) Write it as **proportionality**, then an **equation**: $t \propto 1/d^2$ $t = k/d^2$
2) **Substitute the given values** for the two variables: $25 = k/0.3^2$
3) Rearrange the equation to **find k**: $k = 25 \times 0.3^2 = 2.25$
4) **Put k back in** the formula: $t = 2.25/d^2$
5) **Substitute in the new value** for d: $t = 2.25/0.2^2 = \underline{56.25\text{secs}}$

Student: Date:

Maths Stage 5: Direct & Inverse Proportion

Sheet 10

1) y varies directly with x and y = 8 when x = 3. Find y when x = 18.

2) y varies directly as the square root of x and y = 12 when x =4. Find y when x = 9.

3) Models of cars are made in different sizes. The mass of a model varies directly as the cube of its length. A model 3cm long has mass 1kg. What is the mass of a model 12cm long?

4) On a clear day, the distance that can be seen is directly proportional to the square root of the height above sea level. I can see 10km from a height of 6m. How far could I see from a height of 54m?

Student: Date:

Maths Stage 5: Direct & Inverse Proportion

Sheet 11

1) Electrical resistance of wire varies inversely with the square of its radius. Given that the resistance is 0.4 ohms when the radius is 0.3 cm, find the resistance when radius is 0.45 cm. Answer to 2 decimal places.

2) The time taken to do a piece of work is inversely proportional to the number of students working together. A particular project takes 5 students 10 hours. How long would it have taken 10 students?

3) z is inversely proportional to the square of x .

a) Write down z in terms of x and the constant of proportionality k.

b) If $z = 10$ when $x = 5$, find the value of z when $x = 2$ and of x when $z = 1000$

Student: Date:

Maths Stage 5: Direct & Inverse Proportion

Sheet 12

Show your workings clearly. You may need to use spare paper.

1) The extension of a spring (E) is directly proportional to the force (P) pulling it.
When the force applied is 15 N, the extension is 6cm.
Find the extension when a force of 80 N is applied.

2) R is inversely proportional to the square of S.
When S is 2, R is 50.

a) Calculate S when R = 12.5

b) Calculate R when S is 3.5 (to 1 dp)

3) The volume of a toy is proportional to the cube of is height. A toy that is 2cm tall has a volume of 60 cm^3. What is the volume of a similar toy that is 3 cm taller?

4) A is inversely proportional to $\mathcal{B}$ and $\mathcal{B} = 10$ when $\mathcal{A} = 4$.
Lachlan says that the formula connecting $\mathcal{A}$ and $\mathcal{B}$ is $\mathcal{A} = {}^{40}\!/_{\mathcal{B}}$
Is he right? Explain your answer.

Student: Date:

Maths Stage 5: Sine Rule

Sheet 13

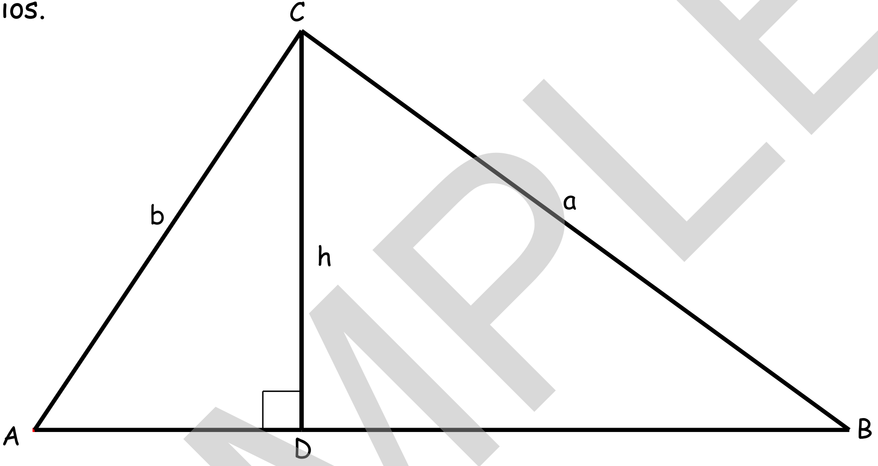
The Sine Rule

Finding unknown angles and sides in right angled triangles is straight forward. We can use sin, cos, tan or Pythagoras. Sadly, not all triangles are right angled. The next method in our armoury is only slightly more difficult. It is called the Sine Rule.

In a triangle *ABC*, $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Proof:

Consider a triangle *ABC* in which there is no right angle – meaning that we cannot use our standard trig ratios.



A line drawn from *C*, perpendicular to *AB*, will divide triangle *ABC* into two right-angled triangles, *CDA* and *CDB*. We can use standard ratios in these.

In $\triangle CDA$ $\sin A = {}^{h}\!/_{b}$ $\Rightarrow$ $h = b \sin A$

In $\triangle CDB$ $\sin B = {}^{h}\!/_{a}$ $\Rightarrow$ $h = a \sin B$

Therefore $a \sin B = b \sin A$

i.e. $$\frac{a}{\sin A} = \frac{b}{\sin B}$$

We could equally well have divided $\triangle ABC$ into two right-angled triangles by drawing the perpendicular from *A* to *BC* (or from *B* to *AC*). This would have led to a similar result.

i.e. $$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Maths Stage 5: Sine Rule

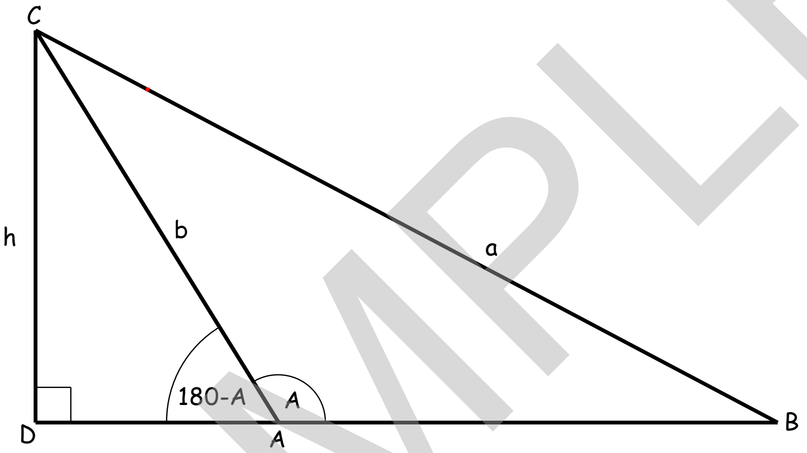
By combining the two results we produce the sine rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This proof is equally valid when ΔABC contains an obtuse angle.

Reminder: When working with a triangle *ABC* the side opposite to $\angle A$ is denoted by *a*, the side opposite to $\angle B$ by *b* and so on.

Suppose that $\angle A$ is obtuse.



This time $h = b \sin(180° - A)$ but, as $\sin(180° - A) = \sin A$, we see that once again $h = b \sin A$.

In all other respects the proof given above is unaltered, showing that the sine rule applies to any triangle.

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This rule is made up of three separate fractions, only two of which are used at a time. We select the two which contain three known quantities and only one unknown.

Note that, when the sine rule is being used to find an unknown angle, it is more conveniently written in the form:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Student: Date:

Maths Stage 5: Sine Rule

Sheet 15

Example: In ΔABC, BC=5cm, A=43° and B=61° . Find AC.

A, B and a are known and b is required so we use $\frac{a}{\sin A} = \frac{b}{\sin B}$

Substituting $\frac{5}{\sin 43} = \frac{b}{\sin 61}$ so $b = \frac{5 \sin 61}{\sin 43} = 6.412$

AC = 6.41 to 3 sig fig.

Now, on a separate sheet of paper, try these questions. Where necessary, give your answers correct to 3 s.f. It will be easier if you do a neat sketch of each triangle. Include this as part of your working.

1) In ΔDEF, DE = 174cm, $\angle D$ = 48° and $\angle F$ = 56°. Find EF.

2) In ΔABC, AB = 9cm, $\angle A$ = 51° and $\angle C$ = 39°. Find BC.

3) In ΔXYZ, $\angle X$ = 27°, YZ = 6.5cm and $\angle Y$ = 73°. Find ZX.

4) In ΔPQR, $\angle R$ = 52°, $\angle Q$ = 79° and PR = 12.7cm. Find PQ.

5) In ΔABC, AC = 9.1cm, $\angle A$ = 59° and $\angle B$ = 62°. Find BC.

6) In ABC, AC = 17cm, $\angle A$ = 105° and $\angle B$ = 33°. Find AB.

And these? Take care! Give your answers to the nearest ° or to 3 sig.fig.

7) In ΔABC, $\angle A$ = 35°, BC = 3cm and AB = 5cm. Find $\angle C$.

8) In ΔXYZ, XZ = 11cm, $\angle Y$ = 41° and YZ = 8cm. Find $\angle X$.

9) In ΔABC, $\angle B$ = 40°, BC = 2.9cm and AC = 6.1cm. Find $\angle A$.

10) In ΔXYZ, XY = 5.7cm, $\angle Y$ = 20° and XZ = 2.3cm. Find $\angle Z$.

11) In ΔABC, $\angle A$ = 29.5°, BC = 36cm and AB = 21cm. Find $\angle C$.

12) In ΔXYZ, XZ = 3.8cm, $\angle Y$ = 54° and YZ = 2.7cm. Find $\angle X$.

13) In ΔABC, $\angle C$ = 33°, AC = 7.1cm and AB = 4.6cm. Find $\angle B$.

14) In ΔXYZ, XY = 9cm, $\angle Z$ = 40° and YZ = 7cm. Find $\angle X$.

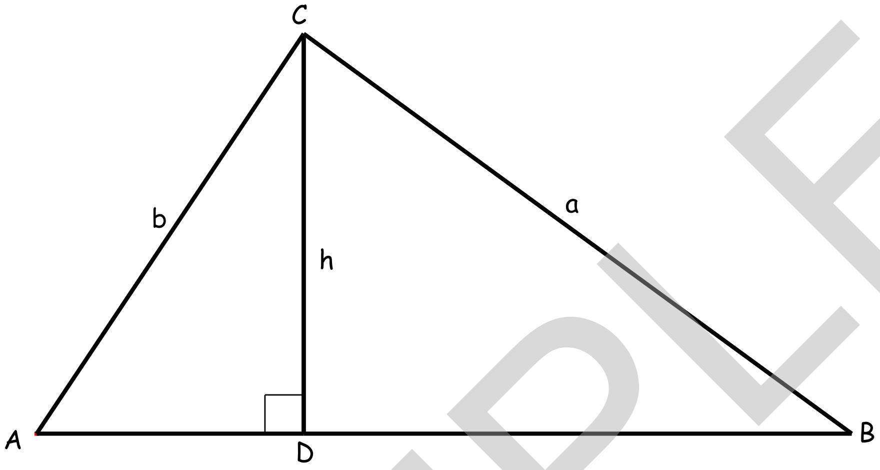
Student: Date:

Maths Stage 5: Sine Rule

Sheet 16

The normal way to calculate area of a triangle is $\frac{1}{2}$ x base x height. Obviously this only works if we know the perpendicular height. Let's think further.

Look at this triangle, in which we know a, b and h.



In triangle ACD, sin A = h/b so h = b sinA

Area of triangle ABC is therefore $\frac{1}{2}$ x c x b sinA

This will normally be expressed with the unknowns in alphabetical order, giving $\frac{1}{2}$ b c sinA

Drawing perpendicular heights from A and B would give similar expressions.

This gives us the general, and easy to remember formula that

Area of triangle ABC = $\frac{1}{2}$ a b sinC

Warning! The area is $\frac{1}{2}$ product of two sides x sine of included angle. You may sometimes have to use sine rule to find the right combinations!

Find the area of each triangle given in these questions, giving your answers to 3 s.f. when necessary. You may use a calculator.

1) In ΔABC, $\angle B = 70°$, BC = 145cm and AB = 108cm.
2) In ΔABC, $\angle A = 62°$, AC = 66cm and AB = 75cm.
3) In ΔPQR, $\angle R = 85°$, QR = 69cm and PR = 49cm.
4) In ΔPQR, $\angle P = 60°$, QR = 12cm and $\angle R = 50°$.

Index notation

Index notation is used to represent powers, for example

a^2 means a × a and here the index is 2

b^3 means b × b × b and here the index is 3

c^4 means c × c × c × c and here the index is 4 etc.

2 x 2 x 2 x 2 = 16 2^4 = 16	• 2 is called the base • 4 is called the index • 16 is called the basic numeral

$$x^n = \underbrace{x \times x \times x \times \ldots\ldots \times x \times x}_{n \text{ factors}} \text{ (where } n \text{ is a positive integer)}$$

For: x^n x is the base

n is the index.

There are several "laws" about index numbers and how they work. We will go through them, one by one. Provided that you understand them then they are easy and you may find that don't even need to memorise them.

Multiplication using indices

$$3^4 \times 3^2 = (3 \times 3 \times 3 \times 3) \times (3 \times 3)$$
$$= 3^6 \quad [=3^{4+2}]$$

$$x^5 \times x^3 = (x \times x \times x \times x \times x) \times (x \times x \times x)$$
$$= x^8 \quad [= x^{5+3}]$$

Law 1: When multiplying terms, *add* the indices: $x^m \times x^n = x^{m+n}$

Division using indices

$$3^5 \div 3^2 = \frac{3x3x3x\cancel{3}x\cancel{3}}{\cancel{3}x\cancel{3}}$$
$$= 3^3[= 3^{5-2}]$$

$$x^4 \div x^3 = \frac{x\,x\,\cancel{x}\,x\,\cancel{x}\,x\,\cancel{x}}{\cancel{x}\,x\,\cancel{x}\,x\,\cancel{x}}$$
$$= x^1[= x^{4-3}]$$

Law 2: When dividing terms, *subtract* the indices: $x^m \div x^n = x^{m-n}$

Powers of indices

$(3^3)^2 = 3^3 \times 3^3$

$= 3^{3+3}$ [Using Law 1]

$= 3^6$ [$= 3^{3 \times 2}$]

$(x^5)^4 = x^5 \times x^5 \times x^5 \times x^5$

$= x^{5+5+5+5}$ [Using Law 1]

$= x^{20}$ [$= x^{5 \times 4}$]

Law 3: For powers of a power, *multiply* the indices: $(x^m)^n = x^{mn}$

If we simplify the division $x^n \div x^n$, using the second law above:

$x^n \div x^n = x^{n-n}$

$= x^0$

But any expression divided by itself must equal 1. $x^n \div x^n = 1$

Therefore x^0 must be equal to 1. $x^0 = 1$

Law 4: $x^0 = 1$

1) Simplify:

a) 4^3 b) 13^5 c) $(-4)^2$

2) Simplify:

a) $3^2 \times 3^5$ b) $x^3 \times x^2$ c) $6m^2n \times mn^4$

3) Simplify:

a) $x^7 \div x^2$ b) $15a^5 \div 3a^2$ c) $20a^3b^2 \div 10ab$

4) Simplify:

a) $(a^4)^2$ b) $(2a^4)^3$ c) $(p^4)^3 \div (p^2)^4$

5) Simplify:

a) 7^0 b) $18x^3 \div 6x^3$ c) $(2y^3)^4 \div (4y^6)^2$

Student: Date:

Maths Stage 5: Indices, Surds & Standard Form

Sheet 19

Negative Indices

All the indices seen so far have been positive integers or zero.

If we had $2^3 \div 2^5$, the answer, according to the second index law, should be 2^{3-5},
ie $2^3 \div 2^5 = 2^{-2}$.

But this could also be written in this way:

$$\frac{2^3}{2^5} = \frac{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1}{\cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times 2 \times 2}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{2^2}$$

So $2^{-2} = \frac{1}{2^2}$

In general, the meaning of a negative index can be summarised by the rules:

$$x^{-m} = \frac{1}{x^m}\ ,\ (x \neq 0)$$

x^{-m} is the reciprocal of x^m, since $x^m \times x^{-m} = 1$.

In English – when you see a minus sign in front of a power, it means that you can rewrite it as 1 over the expression with the minus sign removed. In the example above:

If we have 2^{-2}, the minus sign says write 1 over 2^{-2} with the minus removed so $= \frac{1}{2^2}$

1) Simplify the following:

a) 3^{-2}

b) 5^{-1}

c) $x^7 \times x^{-3}$

d) $6x^2 \div 3x^4$

e) $\left(\frac{1}{4}\right)^{-2}$

f) $\left(\frac{2}{3}\right)^{-3}$

2) Evaluate, using a calculator:

a) 2^{-3}

b) $\left(\frac{1}{3}\right)^{-3}$

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Zero and negative indices

When we are working with numbers, what does it mean to have a zero or negative index? Complete these tables writing answers less than 1 as fractions.

1) Divide each answer by 10 to reduce the power.

Power of 10	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Answer	10000	1000							

÷ 10

As the power of 10 decreases, does the answer decrease?

$10^4 \div 10 = 10^3$
$10^3 \div 10 = 10^2$
$10^2 \div 10 = 10^1$
$10^1 \div 10 = 10^0$
$10^0 \div 10 = 10^{-1}$
Etc….

2) Divide each answer by 5 to reduce the power.

Power of 5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
Answer	625								

÷ 5

As the power of 5 decreases, does the answer decrease?

$5^4 \div 5 = 5^3$
$5^3 \div 5 = 5^2$
$5^2 \div 5 = 5^1$
$5^1 \div 5 = 5^0$
$5^0 \div 5 = 5^{-1}$
Etc….

3) Use the tables above to write true or false for:

a) $10^{-1} = \frac{1}{10}$

b) $5^0 = 1$

c) $5^{-1} = \frac{1}{5}$

d) $10^{-2} = \frac{1}{10^2}$

e) $5^{-3} = \frac{1}{5^3}$

f) $10^0 = 1$

g) $10^{-4} = \frac{1}{10^4}$

h) $5^{-2} = \frac{1}{5^2}$

i) $10^{-3} = \frac{1}{10^3}$

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Complete:

1) $5^2 = 25$ 2) $7^2 = 49$ 3) $2^3 = 8$ 4) $5^3 = 125$

$\sqrt{25} =$ $\sqrt{49} =$ $\sqrt[3]{8} =$ $\sqrt[3]{125} =$

Consider the following:

5) $5^a \times 5^a = 5^{2a}$
If $5^n \times 5^n = 5^1$,
What is the value of n?

6) $8^a \times 8^a = 8^{2a}$
If $8^n \times 8^n = 8^1$,
what is the value of n?

7) $x^a \times x^a = x^{2a}$
If $x^n \times x^n = x^1$,
What is the value of n?

8) $y^a \times y^a = y^{2a}$
If $y^n \times y^n = y^1$,
What is the value of n?

9) $5^a \times 5^a \times 5^a = 5^{3a}$
If $5^n \times 5^n \times 5^n = 5^1$,
What is the value of n?

10) $x^a \times x^a \times x^a = x^{3a}$
If $x^n \times x^n \times x^n = x^1$,
What is the value of n?

11) Simplify the following:

a) $25^{1/2}$ b) $27^{1/3}$ c) $3x^{1/2} x 4x^{1/2}$

d) $(49m^6)^{1/2}$ e) $8^{2/3}$ f) $9^{-3/2}$

12) Evaluate using your calculator:

a) $196^{1/2}$ b) $32^{1/5}$ c) $256^{-1/4}$ d) $125^{-4/3}$

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Sheet 22

Scientific (or Standard) Notation

Use the x^y button on your calculator to answer these questions.

Look for a connection between questions and answers and then fill in the rules at the end of the investigation.

Questions:

1) a) $1.8 \times 10^1 =$ b) $1.8 \times 10^2 =$ c) $1.8 \times 10^3 =$

d) $4.05 \times 10^1 =$ e) $4.05 \times 10^2 =$ f) $4.05 \times 10^3 =$

g) $6.2 \times 10^4 =$ h) $6.2 \times 10^5 =$ i) $6.2 \times 10^6 =$

j) $3.1416 \times 10^2 =$ k) $3.1416 \times 10^3 =$ l) $3.1416 \times 10^4 =$

To multiply by 10^n move the decimal point __________ places to the __________.

2) a) $1.8 \div 10^1 =$ b) $1.8 \div 10^2 =$ c) $1.8 \div 10^3 =$

d) $968.5 \div 10^2 =$ e) $968.5 \div 10^3 =$ f) $968.5 \div 10^4 =$

To divide by 10^n move the decimal point __________ places to the __________.

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The investigation on the previous sheet should have reminded you that:

1) When we *multiply* a decimal by 10, 100 or 1000, we move the decimal point 1, 2 or 3 places to the *right*.

2) When we *divide* a decimal by 10, 100 or 1000, we move the decimal point 1, 2 or 3 places to the *left*.

When expressing numbers in scientific (or standard) notation each number is written as the product of a number between 1 and 10, and a power of 10.

"Scientific notation" is sometimes called "standard notation" or "standard form".

6.1×10^5

This number is written in scientific notation (or standard form).
The first part is between 1 and 10.
The second part is a power of 10.

Scientific notation is useful when writing very large or very small numbers.

Numbers greater than 1

$$5970 = 5.97 \times 10^3$$

To write 5970 in standard form:

- Put a decimal point after the first digit.
- Count the number of places you have to move the decimal point to the left from its original position. This will be the power needed.

1) Express the following in scientific notation.

a) 243 b) 60000 c) 93800000

2) Write the following as a basic numeral.

a) 1.3×10^2 b) 2.431×10^2 c) 4.63×10^7

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Numbers less than 1

$$0.00597 = 5.97 \times 10^{-3}$$

To write 0.00597 in scientific notation:

- Put a decimal point after the first non-zero digit.
- Count the number of places you have moved the decimal point to the right from its original position. This will show the negative number needed as the power of 10.

$$5.97 \times 10^{-3} \text{ is the same as } 5.97 \div 10^{3}$$

1) Express each number in scientific notation.

a) 0.043 b) 0.0000597 c) 0.004

2) Write the basic numeral for:

a) 2.9×10^{-2} b) 9.38×10^{-5} c) 1.004×10^{-3}

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Sheet 25

Surds

Find the value of:

1) $\sqrt{16}$ 2) $\sqrt{9}$ 3) $\sqrt{36}$ 4) $\sqrt{16+9}$ 5) $\sqrt{16}+\sqrt{9}$

6) $\sqrt{16 x 9}$ 7) $\sqrt{16} x \sqrt{9}$ 8) $\sqrt{\frac{36}{9}}$ 9) $\frac{\sqrt{36}}{\sqrt{9}}$ 10) $(\sqrt{16})^2$

Surds are numerical expressions that involve irrational roots. They are irrational numbers.

Surds obey the following rules.

Rule 1 $\sqrt{xy} = \sqrt{x} \text{ x } \sqrt{y}$

Worked examples:

1) $\sqrt{100} = \sqrt{4} x \sqrt{25}$
$= 2 x 5$
$= 10$ (which is true)

2) $\sqrt{27} = \sqrt{9} x \sqrt{3}$
$= 3 x \sqrt{3}$
$= 3\sqrt{3}$

3) $\sqrt{5} x \sqrt{7} = \sqrt{5 x 7}$
$= \sqrt{35}$

Rule 2 $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$ Note: $\sqrt{x}$ *means the positive square root of* x when $x > 0$.

$\sqrt{x} = 0$ when $x = 0$.

Worked examples:

1) $\sqrt{\frac{16}{4}} = \frac{\sqrt{16}}{\sqrt{4}}$
$\sqrt{4} = \frac{4}{2}$
$= 2$ (which is true)

2) $\sqrt{125} \div \sqrt{5} = \sqrt{125 \div 5}$
$= \sqrt{25}$

3) $\sqrt{30} \div \sqrt{5} = \sqrt{30 \div 5}$
$= \sqrt{6}$

Rule 3 $(\sqrt{x})^2 = x$ Note: For $\sqrt{x}$ to exist, x cannot be negative.

Worked examples:

1) $(\sqrt{25})^2 = (5)^2$
$= 25$

2) $(\sqrt{7})^2 = 7$

3) $(3\sqrt{2})^2 = 3^2 x (\sqrt{2})^2$
$= 9 x 2$
$= 18$

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A surd is in its simplest form when the number under the square root sign is as small as possible. To simplify a surd we make use of Rule 1 by expressing the square root as the product of two smaller square roots, one being the root of a square number. Examine the examples below.

Simplify the following surds. The first part of each question has been done.

a) $\sqrt{18} = \sqrt{9} x \sqrt{2}$ b) $\sqrt{75} = \sqrt{25} x \sqrt{3}$ c) $5\sqrt{48} = 5x\sqrt{16} x \sqrt{3}$

Now try these:

Express in terms of the simplest possible surd.

1) $\sqrt{12}$

2) $\sqrt{48}$

3) $\sqrt{200}$

Expand and simplify where this is possible.

4) $\sqrt{3}(2-\sqrt{3})$

5) $\sqrt{2}(5+4\sqrt{2})$

6) $(\sqrt{5}-3)(2\sqrt{5}-4)$

7) $(4+\sqrt{7})(4-\sqrt{7})$

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In general, it is considered better to avoid leaving surds in the denominator when giving an answer. The process of removing them is known as rationalising.

Example

Simplify (rationalise) $\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{2}\sqrt{2}$

Rationalise the denominator, simplifying where possible.

1) $\frac{2}{\sqrt{11}}$

2) $\frac{3\sqrt{2}}{\sqrt{5}}$

3) $\frac{1}{\sqrt{27}}$

4) $\frac{\sqrt{5}}{\sqrt{10}}$

5) $\frac{1}{\sqrt{2}-1}$

6) $\frac{3\sqrt{2}}{5+\sqrt{2}}$