CITY COLLEGE

CITY UNIVERSITY OF NEW YORK

Assignment #2

ME 572: Aerodynamic Design

Fall 2011

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Potential Flow

Submitted By:

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ME 572

1. Consider a Rankine full body formed by a source and a sink with identical strength of $25 m^2/s$ and placed 2 m apart and a uniform flow of 15 m/s.

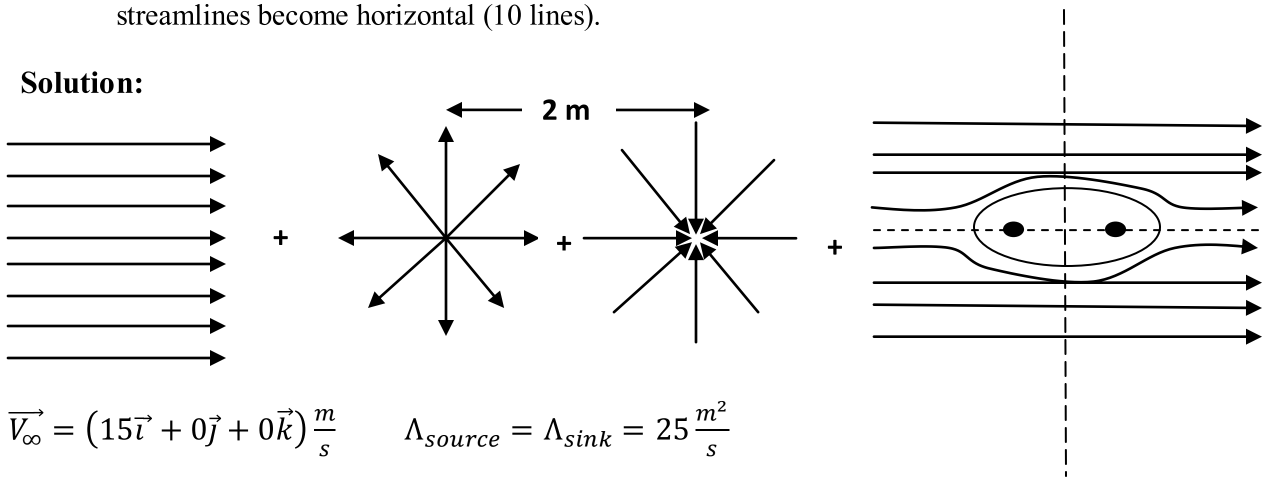
Find:

- The position of the stagnation points.
- Max thickness of the body.

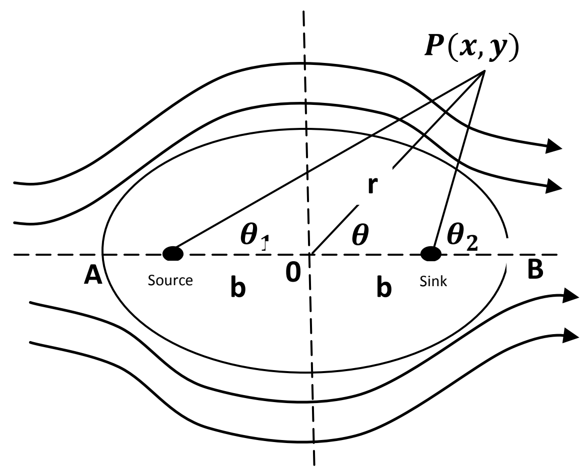
Using MatLab find:

- Pressure and velocity distribution over the body.
- Sketch the streamlines parallel to body up to the point where the streamlines become horizontal (10 lines).

Solution:



$\vec{V_\infty} = (15\vec{i} + 0\vec{j} + 0\vec{k})\frac{m}{s}$ $\Lambda_{source} = \Lambda_{sink} = 25\frac{m^2}{s}$



Step 1: Stream Function

$$a) Uniform\ Flow\ (\psi_1) = V_\infty * r * \sin(\theta)$$

$$b) Source\ (\psi_2) = \frac{\Lambda}{2\pi}(\theta_1)$$

$$c) Sink(\psi_3) = \frac{\Lambda}{2\pi}(\theta_2)$$

Step 2: Superposition of Stream Function

$$\psi = \psi_1 + \psi_2 + \psi_3$$

$$\psi = V_\infty * r * \sin(\theta) + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2) - - - - - - - - (1)$$

Step 3: Velocity

$$\psi = V_\infty * r * \sin(\theta) + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

For Point A:

$$Velocity\ due\ to\ source\ \left(\overrightarrow{V_{A_{source}}}\right) = -\frac{\Lambda}{2\pi(\mathrm{r}-\mathrm{b})}\vec{\mathrm{i}}$$

$$Velocity\ due\ to\ sink\ \left(\overrightarrow{V_{A_{sink}}}\right) = \frac{\Lambda}{2\pi(\mathrm{r}+\mathrm{b})}\vec{\mathrm{i}}$$

Therefore the total velocity at point A is

$$\overrightarrow{V_A} = \overrightarrow{V_\infty} + \left(\overrightarrow{V_{A_{source}}}\right) + \left(\overrightarrow{V_{A_{source}}}\right)$$

$$Let\ A\ be\ the\ stagniation\ point, then\ \overrightarrow{V_A} = 0;$$

$$0 = \overrightarrow{V_\infty} - \frac{\Lambda}{2\pi(\mathrm{r}-\mathrm{b})}\vec{\mathrm{i}} + \frac{\Lambda}{2\pi(\mathrm{r}+\mathrm{b})}\vec{\mathrm{i}}$$

$$Adding\ corresponding\ components$$

$$0 = V_\infty - \frac{\Lambda}{2\pi(\mathrm{r}-\mathrm{b})} + \frac{\Lambda}{2\pi(\mathrm{r}+\mathrm{b})}$$

$$0 = \frac{V_\infty * 2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b}) - \Lambda(\mathrm{r}+\mathrm{b}) + \Lambda(\mathrm{r}-\mathrm{b})}{2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b})}$$

$$0 = \frac{V_\infty * 2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b}) - \Lambda\mathrm{r} - \Lambda\mathrm{b} + \Lambda\mathrm{r} - \Lambda\mathrm{b}}{2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b})}$$

$$0 = \frac{V_\infty * 2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b}) - 2\Lambda\mathrm{b}}{2\pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b})}$$

$$0 = V_\infty * \pi(\mathrm{r}-\mathrm{b})(\mathrm{r}+\mathrm{b}) - \Lambda\mathrm{b}$$

$$0 = V_\infty * \pi * \mathrm{r}^2 - V_\infty * \pi * \mathrm{b}^2 - \Lambda\mathrm{b}$$

$$V_\infty * \pi * \mathrm{b}^2 + \Lambda\mathrm{b} = V_\infty * \pi * \mathrm{r}^2$$

$$\mathrm{r}^2 = \frac{V_\infty * \pi * \mathrm{b}^2 + \Lambda\mathrm{b}}{V_\infty * \pi}$$

$$r = \sqrt{\frac{V_\infty * \pi * b^2}{V_\infty * \pi} + \frac{\Lambda * b}{V_\infty * \pi}}$$

$$r = \sqrt{b^2 + \frac{\Lambda * b}{V_\infty * \pi}} - - - - - - - - - - - (2)$$

Therefore from symmetry we can say that the flow has two stagnation points at A and B such that r_A *and* r_b *is given by equation* (2) @ $\theta, \theta_1, \theta_2$ is 0 point (B) and π point (A).

Part (1a):

Given,

$$V_\infty = 15\frac{m}{s}\ , b = 1m \text{ and } \Lambda = 25\frac{m^2}{s}$$

From Equation (1)

$$r_A = r_B = \sqrt{b^2 + \frac{\Lambda * b}{V_\infty * \pi}} = \sqrt{1 + \frac{25 * 1}{15 * \pi}} = 1.2 \quad m$$ from refrence orgin.

From the code of Appendix (1)

```
Command Window
 Enter the upstream uniform velocity V_0 [m/s] = 15
 Strength of source[m^2/s] = 25
 Strength of sink[m^2/s] = 25
 The distance of seperation between source and sink [m] = 2
>> point0

point0 =

    1.2371

>> point180

point180 =

   -1.2371
```

Part (1b):

The maximum height of the angle is at $\theta = \frac{\pi}{2}$, and $\Psi = 0$.

We have,

$$\frac{h_{max}}{b} = \frac{1}{2}\left[\left(\frac{h_{ma}}{a}\right)^2 - 1\right]\tan^{-1}\left(\frac{2\pi V_{inf} h_{max}}{\lambda}\right) -------- (3)$$

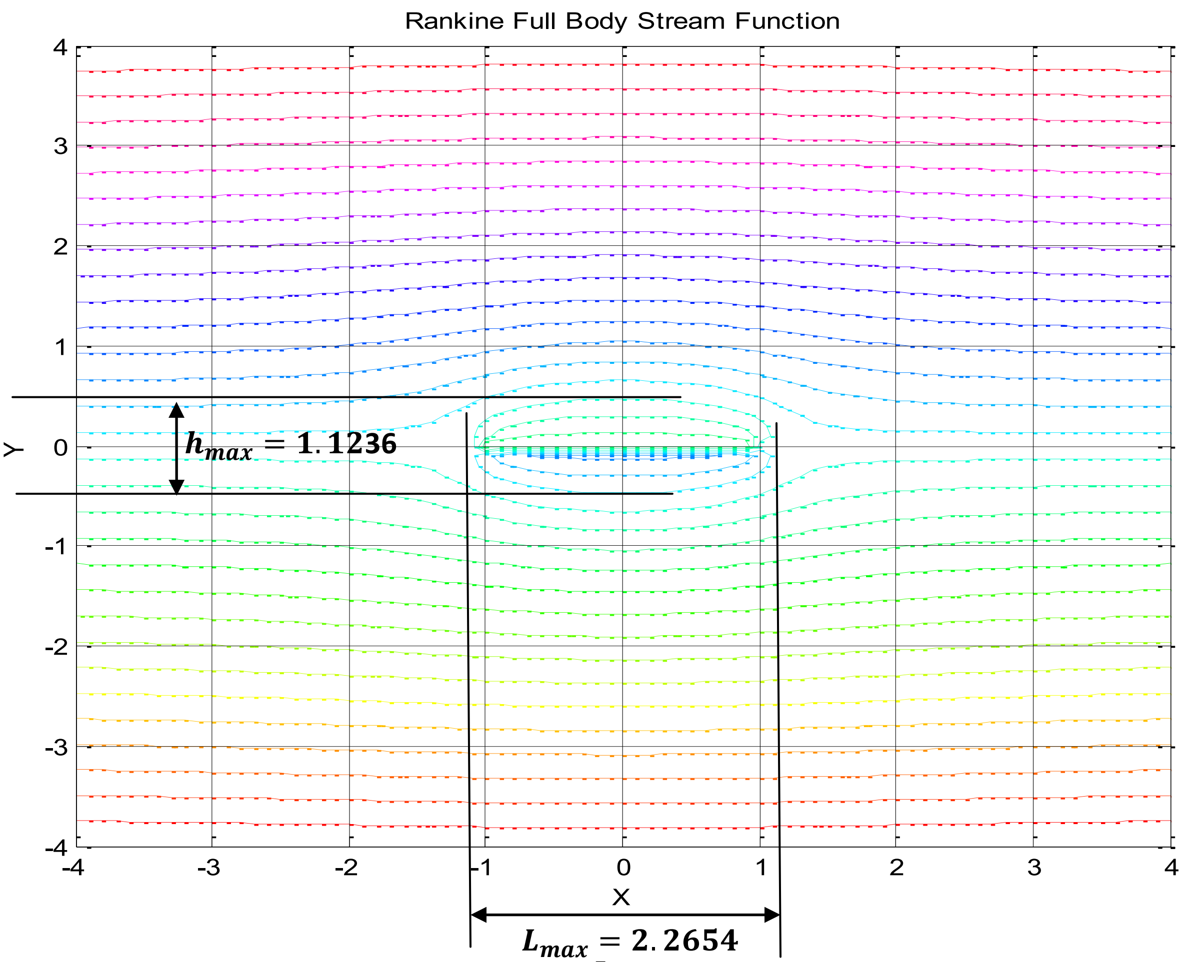
This is a transcendental fucntion, hence from the Newton Raphson n

Hence, the maximum thickness of the body $= 1.12 \quad m$

- Sketch the streamlines parallel to body up to the point where the streamlines become horizontal (10 lines).

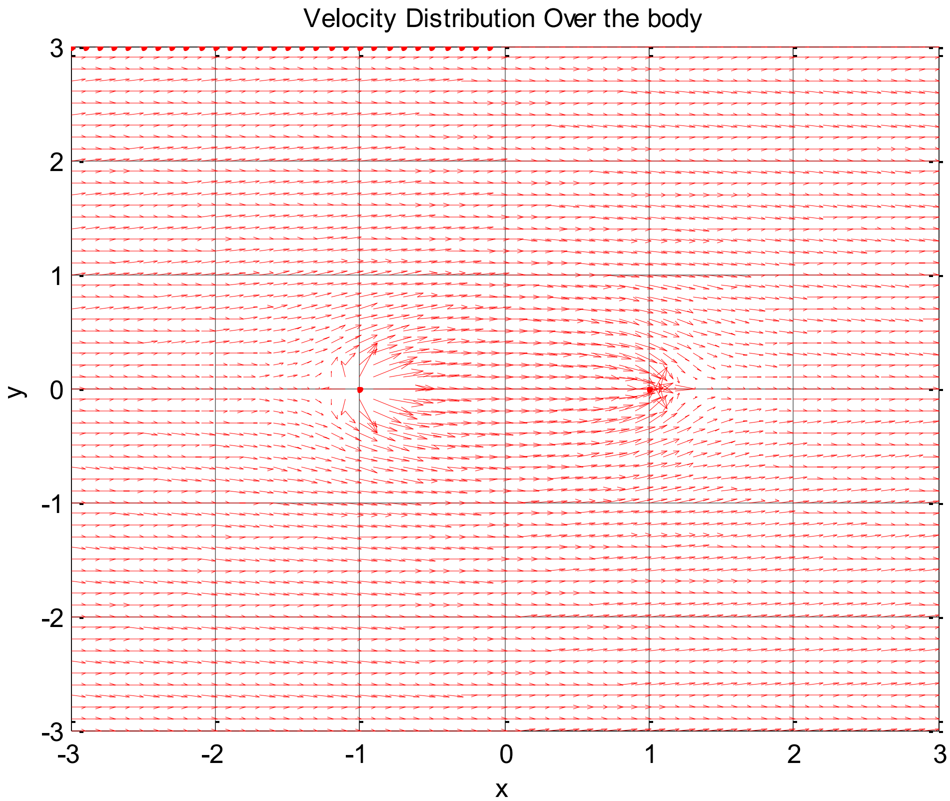
From Appendix (3) and Equation (1)

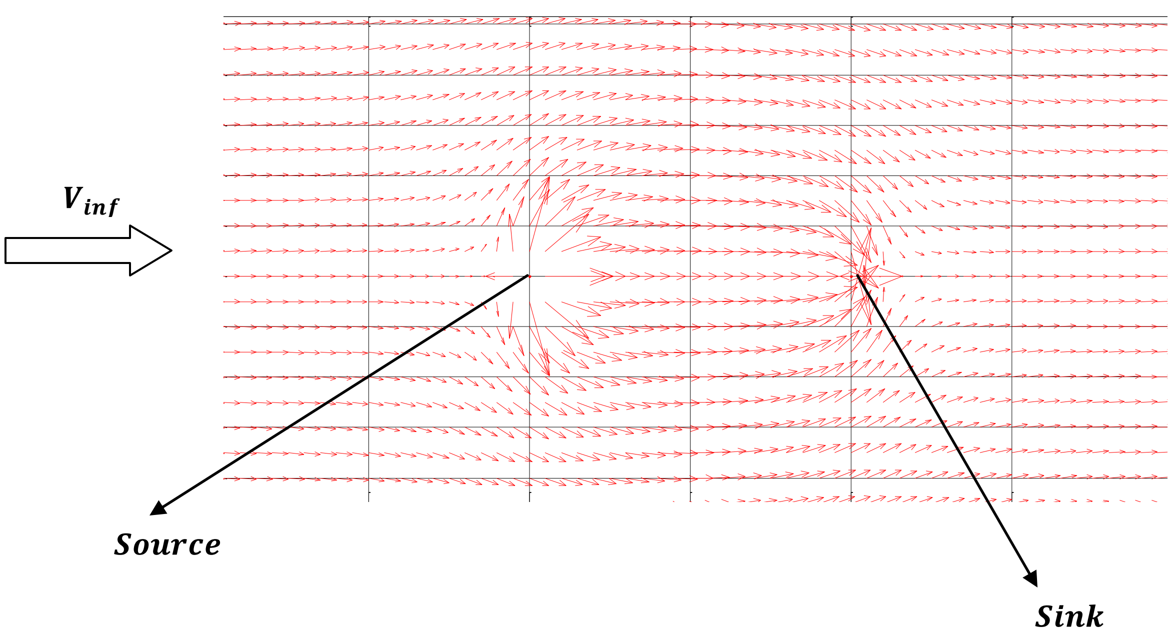
$$\psi = V_\infty * r * \sin(\theta) + \frac{\Lambda}{2\pi}(\theta_1 - \theta_2)$$

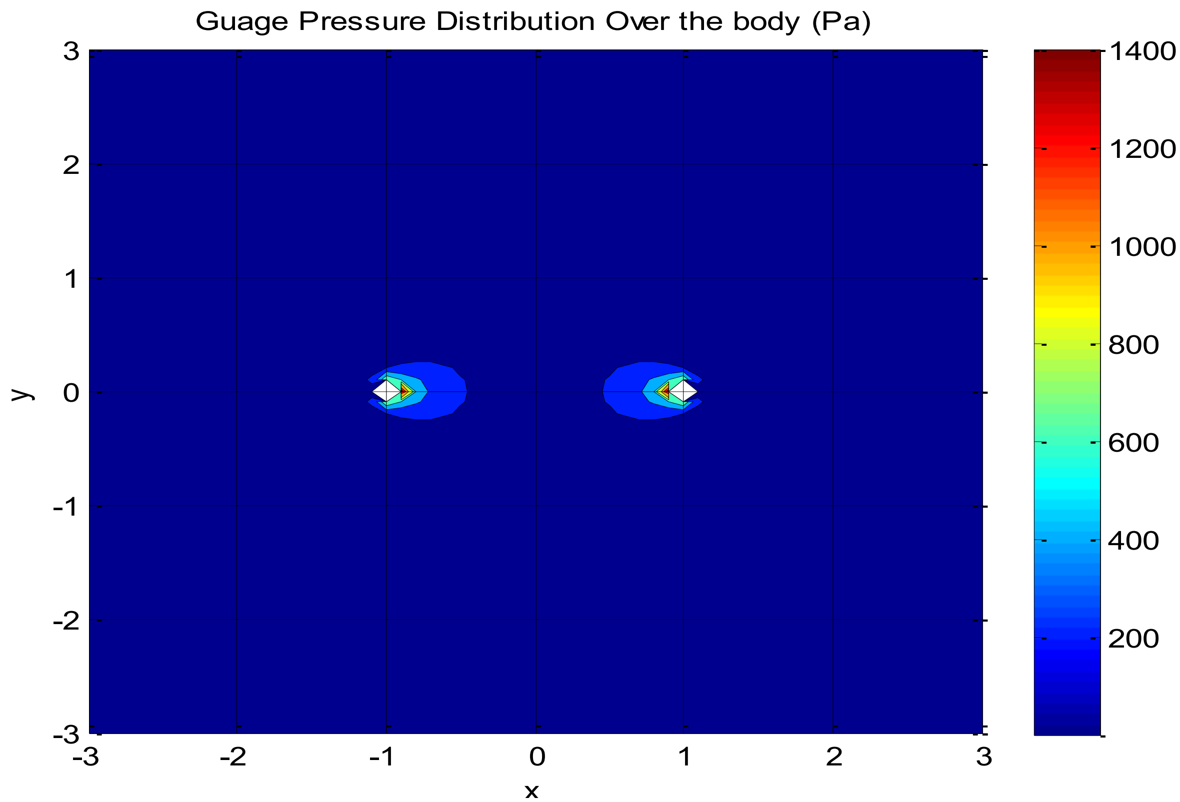


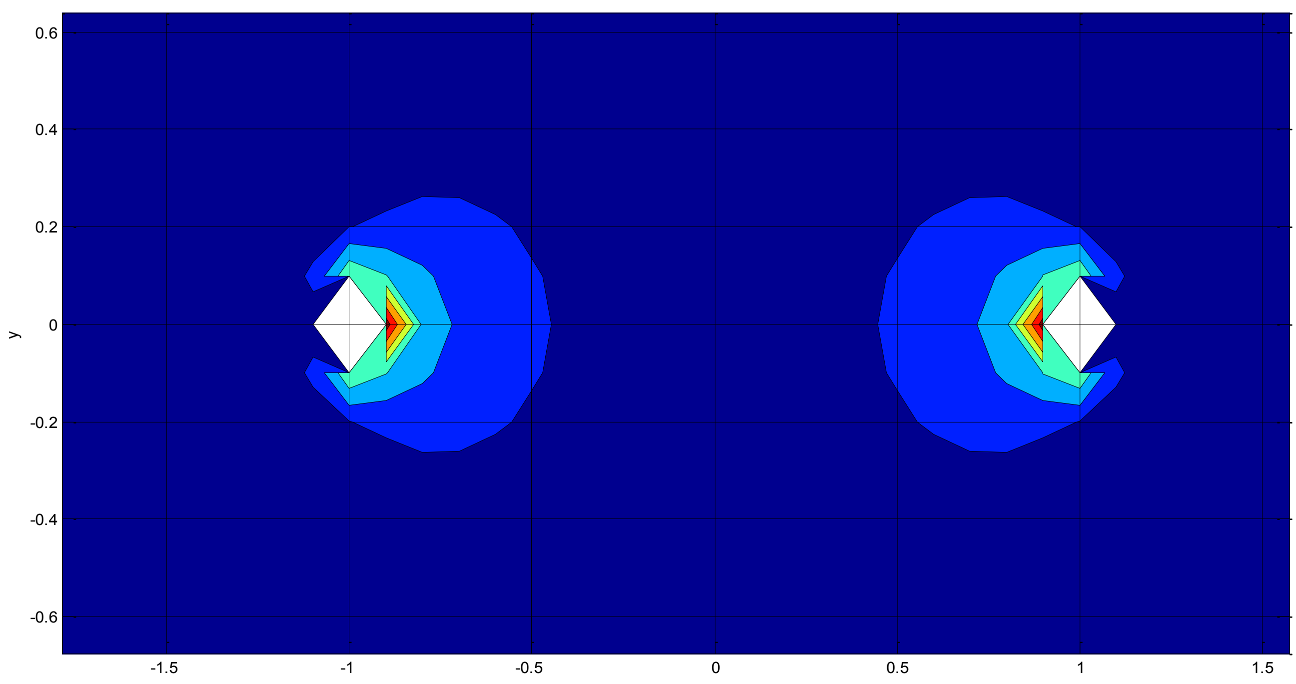
- Pressure and velocity distribution over the body.

  From Appendix (3)






Guage Pressure Distribution Over the body (Pa)
3
2
1
y 0
-1
-2
-3
-3
-2
-1
0
1
2
3
x
1400
1200
1000
800
600
400
200


0.6
0.4
0.2
y 0
-0.2
-0.4
-0.6
-1.5
-1
-0.5
0
0.5
1
1.5
x

2. It's known for a rigid body accelerating (or decelerating) in fluid to generate additional drag force added to the steady state drag, this additional drag can be calculated from the following equation: (for incompressible, 2D flow)

$$F_{D_t} = -\rho I \frac{dU}{dt}$$

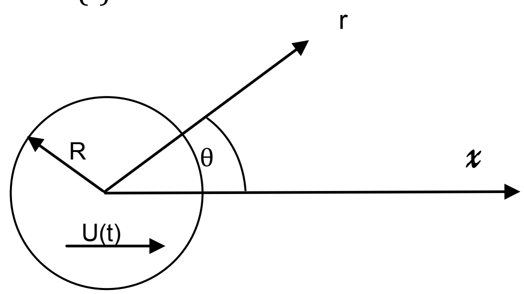
Where ρ is the fluid density, $I = \iiint \frac{u_i}{U}\frac{u_i}{U} dV$ (u_i velocity component in i direction, U is freestream velocity, and V is flow volume (reduced to control volume area in 2D cases). This force is required to move the mass of the body:

$$F = m\frac{dU}{dt}$$

Therefore m which is referred to as the ***added mass***: m=-ρI

Based on the potential flow condition, calculate the added mass for the following:

1. A sphere with radius R is moving with U=U(t)



Let us assume the a sphere of radi -direction velocity U(t), and when a body moves through a fluid, it must push a finite mass of fl

accelerated

ca velocity U(t), the net force m

$$\sum F = (m + m_h).\frac{dU}{dt}$$

where m is the actual mass of the body and m_h is the added mass and is the f

as

Assuming the total kinetic energy of the fl

differential element of the fl is set to be e

bod gi

$$KE_{fluid} = \int \frac{1}{2} * dm * V_{rel}^2 = \frac{1}{2} * m_h U^2$$

For element of fluid mass, in spherical polar coordinates, is

$$dm = \rho * (2 * \pi * r * \sin(\theta))r.dr.d\theta$$

We have potential function of flo sphere in polar co-ordinates s

$$\varphi = \frac{UR^3}{2r^2}\cos(\theta)\ then$$

$$\frac{\partial\varphi}{\partial r} = V_r = -\frac{UR^3}{r^3}\cos(\theta)\ and\ \frac{\partial\varphi}{\partial\theta} = r * V_\theta = \frac{UR^3}{2r}\sin(\theta)$$

$$V_{rel}^2 = V_r^2 + V_\theta^2 = \sqrt{\left(-\frac{UR^3}{r^3}\cos(\theta)\right)^2 + \left(\frac{UR^3}{2r}\sin(\theta)\right)^2}$$

$$= \sqrt{\left(-\frac{U^2R^3}{r^3}\cos(\theta)\right)^2 + \left(\frac{UR^3}{2r}\sin(\theta)\right)^2}$$

$$= \frac{1}{2} * \sqrt{\left(-\frac{U^2 * R^6 * (-4 * cos(\theta)^2 - r^2 + r^2 * cos(\theta)^2)}{r^6}\right)}$$

Now s the

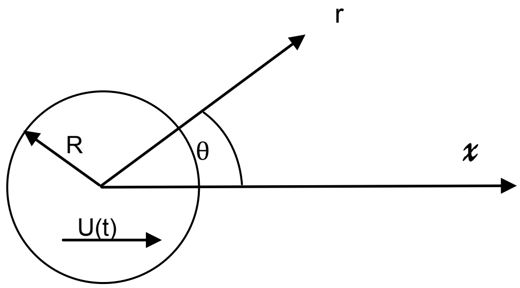
$$\int \frac{1}{2} * dm * V_{rel}^2 = \frac{1}{2} * m_h U^2$$

$$\int\int \frac{1}{2} * \rho * (2 * \pi * r * \sin(\theta)) * r * \frac{1}{2} * \left(-\frac{U^2 * R^6 * \left(-4 * cos(\theta)^2 - r^2 + r^2 * cos(\theta)^2\right)}{r^6}\right) drd\theta - \frac{1}{2} * m_h U^2 = 0$$

$$\int \rho * \pi * \sin(\theta) * \left(-\frac{R^6 * \left(-4 * cos(\theta)^2 - r^2 + r^2 * cos(\theta)^2\right)}{r^4}\right) drd\theta - m_h = 0$$

Solving the equation symbolically in Maple $= m_h = \frac{2}{3}\rho\pi R^3$

2. A cylinder with radius R is moving with U=U(t)



Similarly, 2D cylinder mo t

$$m_h = \frac{1}{U^2}\int\int\left(\left(\frac{Ua^2}{r^2} * \sin(2\theta)\right)^2 + \left(\frac{Ua^2}{r^2} * \cos(2\theta)\right)^2\right) r dr d\theta$$

Solving the equation symbolically in Maple = $m_h = \rho\pi R^2$

Appendix (1)

```
clear all
close all
clc
```

```
V_inf = input(' Enter the upstream uniform velocity V_0 [m/s] = ');
Lambda1 = input(' Strength of source[m^2/s] = ');
Lambda2 = input(' Strength of sink[m^2/s] = ');
d = input(' The distance of seperation between source and sink [m] = ');


% Finding the stagnation points
n=d/2;
root_1=(1/4)*(-Lambda1+Lambda2+sqrt(Lambda1^2-
2*Lambda1*Lambda2+Lambda2^2+8*V_inf*pi*n*Lambda2+16*V_inf^2*pi^2*n^2+8*V_inf*
pi*n*Lambda1))/(V_inf*pi);
root_2=(-1/4)*(-Lambda1-Lambda2+sqrt(Lambda1^2-
2*Lambda1*Lambda2+Lambda2^2+8*V_inf*pi*n*Lambda2+16*V_inf^2*pi^2*n^2+8*V_inf*
pi*n*Lambda1))/(V_inf*pi);
if root_1<0
    a=root_2;
    point0=a;
    point180=-a;
else
    a=root_1;
    point0=a;
    point180=-a;
end
  disp('The stagnation point are at: '), disp(point0), disp('and'),
disp(point180);
```

Appendix (2)

```
% Finding the maximum thickness points
 i=1;
 r(i,1)=input('Enter initial guess: ');
 error_max=input('Enter Maximum Error: ');
while(1)
    a=d/2;
   [h dhdr]=maxi_heigth(V_inf,d,Lambda1,r(i,1));
    r(i+1,1)=r(i,1)-h/dhdr;
    error=abs(r(i+1,1)-r(i,1));
    if error<error_max
        result=2*r(i+1,1);
      break,
    end
     i=i+1;
end

   disp('The maximum thickness of Rankine Oval is: '), disp(result);
```

Appendix (3)

```
    % Creating Mesh Grid
    X=-3:.1:3;
    Y=X;
    P_atm=0;
    rho=1;
    [x,y] = meshgrid(X,Y);
```

```
    theta1=atan2(y,(x+n));
    theta2=atan2(y,(x-n));
    psi=V_inf.*y+(0.5.*Lambda1.*theta1)./pi-(0.5.*Lambda2.*theta2)./pi;
    u=V_inf+(0.5.*Lambda1.*(x+n))./(pi.*((x+n).^2+y.^2))-(0.5.*Lambda1.*(x-
n))./(pi.*((x-n).^2+y.^2));
    v=(0.5.*Lambda1.*y)./(pi.*((x+n).^2+y.^2))-(0.5.*Lambda2.*y)./(pi.*((x-
n).^2+y.^2));
    px=0.5.*rho.*(u.^2-V_inf.^2);
    py=-0.5.*rho.*v.^2;
    pabs=sqrt(px.^2+py.^2);

    figure(1)
    contour(x,y,psi,30);
    grid on
    xlabel('X')
    ylabel('Y')
    title('Rankine Full Body Stream Function')
    xlim([-3 3])
    ylim([-3 3])

    figure(2)
    quiver(x,y,u,v,3,'r');
    grid on
    xlabel('x');
    ylabel('y');
    xlim([-3 3])
    ylim([-3 3])
    title('Velocity Distribution Over the body (m/s)')

    figure(3)
    contourf(x,y,pabs);
    view(0,90)
    grid on
    xlabel('x');
    ylabel('y');
    xlim([-3 3])
    ylim([-3 3])
    title('Guage Pressure Distribution Over the body (Pa)')
    colorbar;
```