CITY COLLEGE

CITY UNIVERSITY OF NEW YORK

Assignment #3

ME 572: Aerodynamic Design

Fall 2011

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Vortex Sheet and Vortex Panel

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November 7, 2011

Nomenclature

$X = x - coordinate\ of\ position\ along\ the\ Airfoil$

$Y = y - coordinate\ of\ position\ along\ the\ Airfoil$

$V_\infty = Unifrom\ Upstream\ velocity\ in\ x - direction$

$\rho = density$

$p = pressure$

$\kappa = effective\ \ vortex\ sheet\ stength$

$\gamma = local\ \ vortex\ stength$

Problem Statement

There is a pressure difference across the vortex sheet proportional to the local vortex strength.

$$\Delta p = \rho * V_\infty * \kappa$$

Introduction

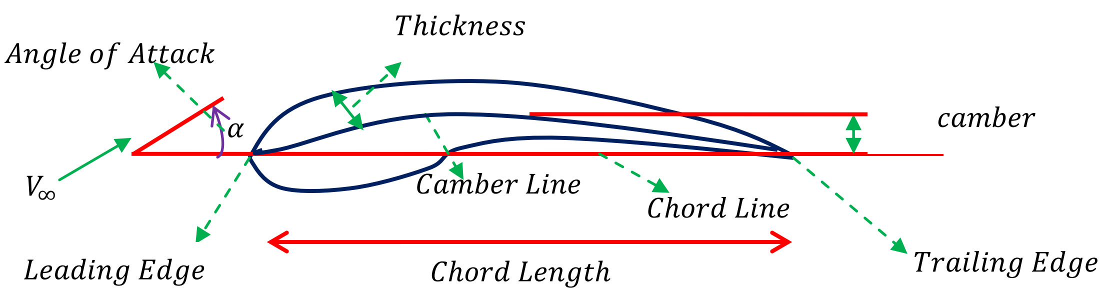


Figure 1.0 Airfoil

An airfoil is a section of a wing, as depicted in figure 1.0 as show above and the shape of the airfoil is determined by the following geometric parameters.

- The *chord line*, defined as the straight line connecting the leading edge to the trailing edge.
- The *chord*, defined as the distance from the leading edge to the trailing edge.
- The *camber line*, defined as the locus of points located halfway between the upper and lower surface of the airfoil.
- The *camber*, defined as the maximum distance of the camber line from the chord line. If the camber is zero, the airfoil is symmetric.
- The angle subtended between the incoming wind and the chord line, α, is defined as the *angle of attack*.

Proof

The vortex is a flow wherein the fluid circulates about a point. The angular velocity decreases in a logrithmetic fashion as the distance from the point increases. As such the origin of the vortex is a singular point, that is, one where the fluid velocity is infinite. For a vortex,

$$\psi = \frac{\kappa}{2\pi} * \ln(r) ---------------------(1)$$

Generalized expression such that the stream function associated with the vortex sheet is given by

$$\psi^{VS}(x,y) = \frac{1}{4\pi}\oint \ln[(x-x')^2 + (y-y')^2)\,\gamma(x').dl' ----------(2)$$

where $dl' = (dx'^2 + dy'^2)^{\frac{1}{2}}$ is the differential arc length around the curve segments.

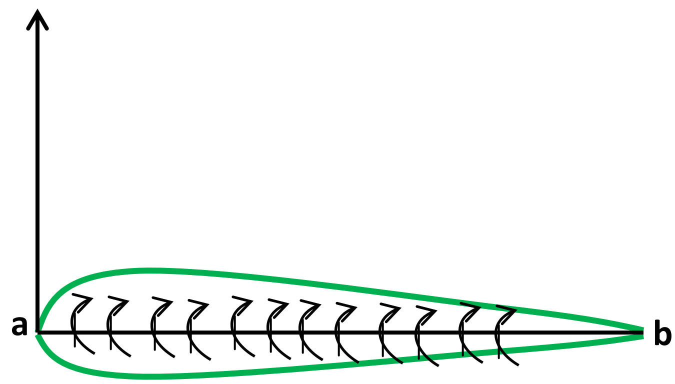


Figure 2.0 Linear camber line with multiple vortex points along the global (y-x) vortex sheet.

For a thin airfoil, assuming the chamber line and chord line are collinear along x-axis as show in the figure 2.0. Since the both side of airfoil is close to x-axis, the corresponding line integral where the leading edge is at point (a) and trailing edge is at point (b). As we can see from the figure than both side of the airfoil are close to x-axis, the corresponding line integral can be approximated with integral with respect $to\ x\ from\ x = 0\ to\ x = c$. Setting $y' = 0$, tracing the airfoil in the clockwise direction beginning at the trailing edge and noting that on the upper side of the airfoil $dl' = dx'$ while on the lower side $dl' = dx'$, we find

$$\psi^{VS}(x,y) = \frac{1}{4\pi}\int_0^C \log[(x-x')^2 + y^2)\,(\gamma^+ - \gamma^-)(x').dx' ---------(3)$$

where the subscript + and – denote, the upper and lower side respectively.

Defining $\kappa \equiv \gamma^{+} - \gamma^{-}$, we obtain a representation in terms of an effective flat vortex sheet subteneded between the ;eading and trailing edge, such that

$$\psi^{VS}(x,y) \simeq \frac{1}{4\pi}\int_{0}^{C} \log[(x-x')^2 + y^2)\,\kappa(x').\,dx' - - - - - - - - - (4)$$

and

$$\phi^{VS}(x,y) \simeq -\frac{1}{2\pi}\int_{0}^{C} \tan^{-1}\left(\frac{y}{x-x'}\right)\kappa(x').\,dx' - - - - - - - - - - (5)$$

As a preliminary, we consider the velocity induced by the vortex sheet in the upper and lower side of figure (2). To begin we consider the limit of the velocity potential as the evaluation point $X = (x,y)$ approaches the vortex sheet from upper side; that is $y \longrightarrow 0+$ with $0 < x < c$. In this limit, the inverse tangent function of equation (4) is 0 $when\ x' < x\ or\ \pi\ when\ x' > x$. Consequently, the potential function equation takes the value,

$$\phi^{VS}(x, y \to 0+) = -\frac{1}{2}\int_{x}^{C} \kappa(x').\,dx' - - - - - - - - - - - - - - (5)$$

Differentiating both sides of equation with respect to x, we find that the x velocity component is given by,

$$U_x^{\ VS}\,(x, y \to 0+) = \frac{1}{2}\kappa(x) - - - - - - - - - - - - - - (6)$$

Similarly for the lower side, we find

$$U_x^{\ VS}\,(x, y \to 0-) = -\frac{1}{2}\kappa(x) - - - - - - - - - - - - - - - (7)$$

Therefore the pressure difference on the either side of the vortex sheet representing the thin airfoil,

$$\Delta p \equiv\ p\,(x, y \to 0-) - p\,(x, y \to 0+)$$

We can compute this using the Bernoulli equation,

$$\Delta p \equiv \left[p_\infty - \frac{1}{2}\rho\left(V_\infty - \frac{1}{2}\kappa(x)\right)^2\right] - \left[p_\infty - \frac{1}{2}\rho\left(V_\infty + \frac{1}{2}\kappa(x)\right)^2\right]$$

$$\Delta p \equiv \left[\cancel{p_\infty} - \frac{1}{2}\rho\left(V_\infty - \frac{1}{2}\kappa(x)\right)^2 \cancel{- p_\infty} + \frac{1}{2}\rho\left(V_\infty + \frac{1}{2}\kappa(x)\right)^2\right]$$

$$\Delta p \equiv \left[-\frac{1}{2}\rho\left(V_\infty^2 - 2 * V_\infty * \frac{1}{2}\kappa(x) + \frac{1}{4}\kappa(x)^2\right) + \frac{1}{2}\rho\left(V_\infty^2 + 2 * V_\infty * \frac{1}{2}\kappa(x) + \frac{1}{4}\kappa(x)^2\right)\right]$$

$$\Delta p \equiv \left[\left(\cancel{-\frac{1}{2}\rho V_\infty^2} + \frac{1}{2}\rho * 2 * V_\infty * \frac{1}{2}\kappa(x) \cancel{- \frac{1}{2}\rho * \frac{1}{4}\kappa(x)^2} \cancel{+ \frac{1}{2}\rho V_\infty^2} + \frac{1}{2}\rho * 2 * V_\infty * \frac{1}{2}\kappa(x) \cancel{+ \frac{1}{2}\rho * \frac{1}{4}\kappa(x)^2}\right)\right]$$

$$\Delta p \equiv \left[\left(\frac{1}{\cancel{2}}\rho * \cancel{2} * V_\infty * \frac{1}{2}\kappa(x) + \frac{1}{\cancel{2}}\rho * \cancel{2} * V_\infty * \frac{1}{2}\kappa(x)\right)\right]$$

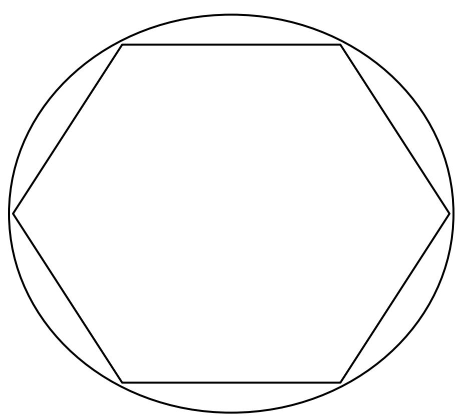
$$\Delta p \equiv \left[\left(\rho * V_\infty * \frac{1}{2}\kappa(x) + \rho * V_\infty * \frac{1}{2}\kappa(x)\right)\right]$$

$$\Delta p \equiv \left[\left(\rho * V_\infty * \kappa(x)\right)\right]$$

$$\boldsymbol{\Delta p = \rho * V_\infty * \kappa}$$

Problem Statement

Calculate the pressure coefficient distribution around circular cylinder using source panel method.



We know for non-lifting uniform flow over the Circular cylinder is given by

$$\boldsymbol{C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 = 1 - 4 * sin^2(\theta)}$$

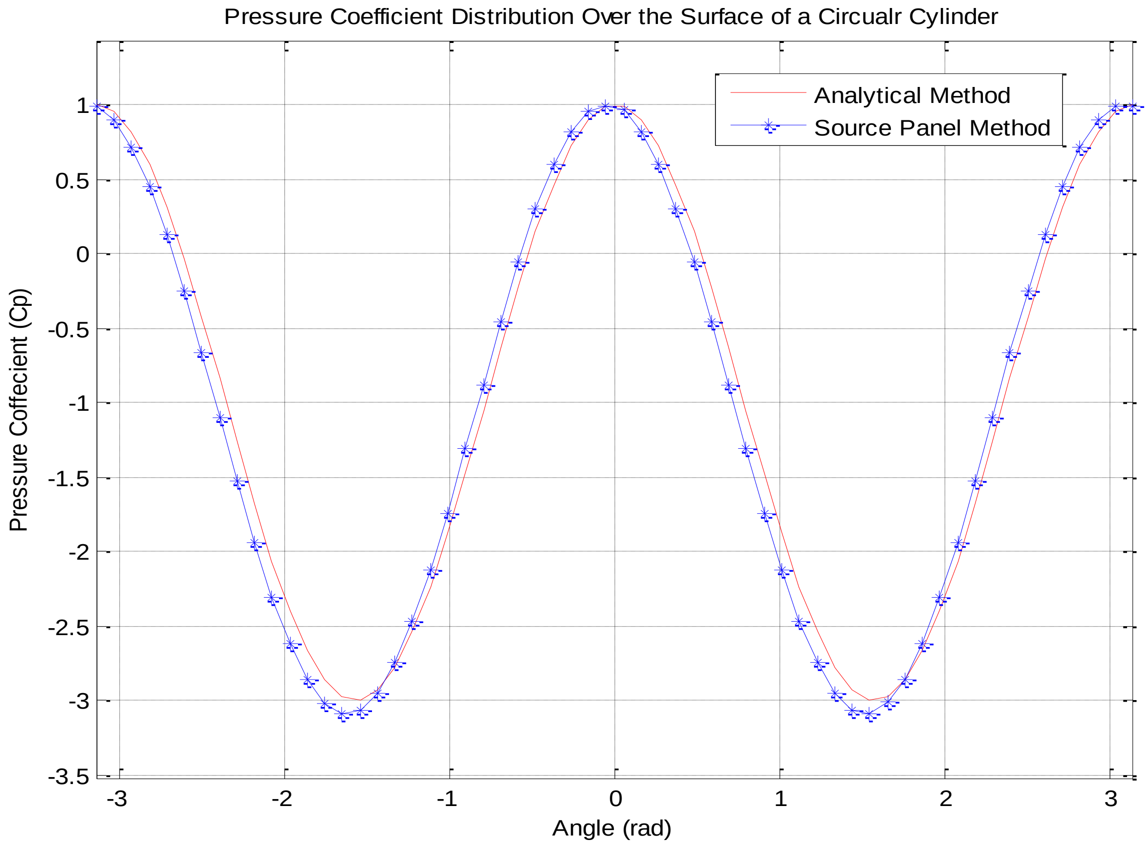
MatLab Code

```
clear all
close all
clc

% Numerical Solution for Flow over the Circular Cylinder Using Source Panel
Method
cycl=load('cylinder.txt');
cycl_x=cycl(:,1);
cycl_y=cycl(:,2);


for i=1:length(cycl_x)-1
    pt1(i,1) = cycl_x(i,1);
    pt1(i,2) = cycl_y(i,1);
    pt2(i,1) = cycl_x(i+1,1);
    pt2(i,2) = cycl_y(i+1,1);
end

for i=1:length(cycl_x)-1
    dx = pt2(i,1)-pt1(i,1);
    dy = pt2(i,2)-pt1(i,2);
    dl(i) = sqrt( (pt2(i,1)-pt1(i,1))^2 ... % panel length
        + (pt2(i,2)-pt1(i,2))^2);
    th(i) = atan2(dy,dx);
    tnx(i) = cos(th(i)); % tangential vector
    tny(i) = sin(th(i));
    vnx(i) = -tny(i); % normal vector points into flow
    vny(i) = tnx(i);
end

for i=1:length(cycl_x)-1
    co(i,1) = 0.5*(pt1(i,1)+pt2(i,1));
    co(i,2) = 0.5*(pt1(i,2)+pt2(i,2));
end

for i=1:length(cycl_x)-1
    for j=1:length(cycl_x)-1
        xt = co(i,1)-pt1(j,1); % shift collocation point
        yt = co(i,2)-pt1(j,2);
        x = xt*tnx(j)+yt*tny(j); % and rotate
        y = -xt*tny(j)+yt*tnx(j);
        x1 = 0.0; y1 = 0.0;
        x2t = pt2(j,1)-pt1(j,1); % shift second-end point
        y2t = pt2(j,2)-pt1(j,2);
        x2 = x2t*tnx(j) + y2t*tny(j); % and rotate y2 = 0.0;
        r1 = sqrt(x*x+y*y);
        r2 = sqrt((x-x2)*(x-x2)+y*y);
        th1 = atan2(y,x);
        th2 = atan2(y,x-x2);
        if(i==j) % self-induced velocity
```

```
            ax1 = 0.5*(x/x2-1.0);
            ay1 = 1.0/(2*pi);
            ax2 =-0.5*x/x2;
            ay2 =-1.0/(2*pi);
        else
            dth = th2-th1;
            rrt = r2/r1;
            rrtl = log(rrt);
            fcc = 1.0/(2*pi*x2);
            ax1 = fcc*( y*rrtl + (x-x2)*dth );
            ay1 = fcc*((x-x2)*rrtl - y*dth + x2);
            ax2 = -fcc*(y*rrtl + x*dth );
            ay2 = -fcc*(x*rrtl - y*dth + x2);
        end
        ux1 = ax1*tnx(j) - ay1*tny(j);
        uy1 = ax1*tny(j) + ay1*tnx(j);
        ux2 = ax2*tnx(j) - ay2*tny(j);
        uy2 = ax2*tny(j) + ay2*tnx(j);

        if(j==1)
            a(i,1)= ux1*vnx(i) + uy1*vny(i);
            holda = ux2*vnx(i) + uy2*vny(i); % hold for the next panel
        elseif(j==length(cycl_x)-1)
            a(i,length(cycl_x)-1) = ux1*vnx(i) + uy1*vny(i) + holda;
            a(i,length(cycl_x)-1+1) = ux2*vnx(i) + uy2*vny(i);
        else
            a(i,j)= ux1*vnx(i) + uy1*vny(i) + holda;
            holda = ux2*vnx(i) + uy2*vny(i); % hold for the next panel
        end
        if(j==1)
            b(i,1)= ux1*tnx(i) + uy1*tny(i);
            holdb = ux2*tnx(i) + uy2*tny(i);
        elseif(j==length(cycl_x)-1)
            b(i,length(cycl_x)-1) = ux1*tnx(i) + uy1*tny(i) + holdb;
            b(i,length(cycl_x)-1+1) = ux2*tnx(i) + uy2*tny(i);
        else
            b(i,j)= ux1*tnx(i) + uy1*tny(i) + holdb;
            holdb = ux2*tnx(i) + uy2*tny(i); % hold for the next panel
        end
    end
end

%--------------------------------
% Add the Kutta condition
% expressed by the length(cycl_x) equation:
% The strength of the sheet at the first point
% of the first panel is equal and opposite
% to the strength of the sheet at the last point
% of the last panel,
% so that the mean value vanishes
%--------------------------------
a(length(cycl_x),1) = 1.0;
a(length(cycl_x),length(cycl_x)) = 1.0;
%--------------------
% set right-hand side
%--------------------
```

```
alpha = 0.0; % angle of attack in degrees
Umag = 1.0; % incident velocity
al = alpha*pi/180.0;
cal = cos(al); sal = sin(al);
Ux = Umag*cal; Uy = Umag*sal;

for i=1:length(cycl_x)-1
rhs(i) = Ux*vnx(i)+Uy*vny(i);
end
rhs(length(cycl_x))=0.0;

%-----------------------
% solve the linear system
%-----------------------
gamma = rhs/a';
%---------------------------------
% compute c p and the circulation
%---------------------------------

for i=1:length(cycl_x)-1
velt = Ux*tnx(i)+Uy*tny(i); % tangential velocity
for j=1:length(cycl_x)
velt = velt + b(i,j)*gamma(j);
end
cp(i) = 1.0-(velt/Umag)^2; % press coeff
end
cp(length(cycl_x)-1+1)=cp(1);

% Theoritical Solution for Flow over the Circular Cylinder
theta=-pi:2*pi/59:pi;
cp_t=1-4*(sin(theta)).^2;
plot(theta,cp_t,'-r',theta,cp,'-*b')
grid on
axis equal
xlabel('Angle (rad)')
ylabel('Pressure Coffecient (Cp)')
title('Pressure Coefficient Distribution Over the Surface of a Circualr
Cylinder')
legend('Analytical Method', 'Source Panel Method')
```

Results


Pressure Coefficient Distribution Over the Surface of a Circualr Cylinder
Analytical Method
Source Panel Method
Pressure Coffecient (Cp)
Angle (rad)
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-3
-2
-1
0
1
2
3

Reference

- Anderson, John D. "Fundamentals of Aerodynamics" McGraw Hill. 2001: 3rd edition.
- Pozrikidis, C. "Fluid Dynamics: Theory, Computation and Numerical Simulation", Springer Science Business Media, LLC 2009. : 2nd edition.

Appendix

Cylinder Data Point

1.0000	0	0	0
0.9972	0.0531	0.0064	-0.0795
0.9887	0.1057	0.0176	-0.1316
0.9747	0.1570	0.0343	-0.1821
0.9553	0.2066	0.0563	-0.2305
0.9308	0.2538	0.0833	-0.2764
0.9014	0.2982	0.1151	-0.3191
0.8674	0.3392	0.1512	-0.3582
0.8293	0.3763	0.1912	-0.3933
0.7874	0.4092	0.2348	-0.4239
0.7423	0.4374	0.2813	-0.4497
0.6944	0.4607	0.3304	-0.4704
0.6443	0.4787	0.3813	-0.4857
0.5926	0.4913	0.4336	-0.4956
0.5399	0.4984	0.4867	-0.4998
0.4867	0.4998	0.5399	-0.4984
0.4336	0.4956	0.5926	-0.4913
0.3813	0.4857	0.6443	-0.4787
0.3304	0.4704	0.6944	-0.4607
0.2813	0.4497	0.7423	-0.4374
0.2348	0.4239	0.7874	-0.4092
0.1912	0.3933	0.8293	-0.3763
0.1512	0.3582	0.8674	-0.3392
0.1151	0.3191	0.9014	-0.2982
0.0833	0.2764	0.9308	-0.2538
0.0563	0.2305	0.9553	-0.2066
0.0343	0.1821	0.9747	-0.1570
0.0176	0.1316	0.9887	-0.1057
0.0064	0.0795	0.9972	-0.0531
0.0007	0.0266	1.0000	-0.0000