1: Essentials of Geometry

Main Ideas
In this chapter students will name and sketch geometric figures, use postulates to identify congruent segments, find lengths of segments in the coordinate plane, and find the midpoint of a segment. Students will also name, measure and classify angles, identify complementary and supplementary angles, and classify polygons.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

<table>
<thead>
<tr>
<th>How student answers the exercises</th>
<th>What to assign from Skills Readiness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Any of Exs. 3–6 answered incorrectly</td>
<td>Skill 54 Simplify absolute value expressions</td>
</tr>
<tr>
<td>Any of Exs. 7–10 answered incorrectly</td>
<td>Skill 60 Evaluate variable expressions</td>
</tr>
<tr>
<td>Any of Exs. 11–16 answered incorrectly</td>
<td>Skill 69 Solve equations</td>
</tr>
<tr>
<td>All exercises answered correctly</td>
<td>Chapter Enrichment</td>
</tr>
</tbody>
</table>

Previously, you learned the following skills, which you'll use in this chapter: finding measures, simplifying and evaluating expressions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK
Copy and complete the statement.

1. The distance around a polygon is called its ___, and the distance around a circle is called its ___.  perimeter, circumference

2. The number of square units covered by a figure is called its ___. area

SKILLS AND ALGEBRA CHECK
Simplify the expression.

3. \(|4 - 6| 2\)
4. \(|3 - 11| 8\)
5. \(|-4 + 5| 1\)
6. \(|-8 - 10| 18\)

Evaluate the expression when \(x = 2\).

7. \(5x - 10\)
8. \(20 - 8x\)
9. \(-18 + 3x - 12\)
10. \(-5x - 4 + 2x - 10\)

Solve the equation.

11. \(2 \div 4 = -2x - 13\)
12. \(8x - 12 = 60\)
13. \(2y - 5 + 7y = -32 - 3\)
14. \(6p + 11 + 3p = -7 - 2\)
15. \(4 + \frac{m}{7} = 10\)
16. \(5n - 8 = 47\)

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

The Common Core Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Opportunities to develop these practices are integrated throughout this program. Some examples are provided below.

1. Make sense of problems and persevere in solving them. Pages 23, 49, 50.
2. Reason abstractly and quantitatively. Pages 9, 19, 40, 44.
3. Construct viable arguments and critique the reasoning of others. Pages 12, 30, 39, 45.
4. Model with mathematics. Pages 10, 21, 27, 36, 44.

5. Use appropriate tools strategically. Pages 15, 27, 33, 48.
6. Attend to precision. Pages 5, 12, 19, 23.
7. Look for and make use of structure. Pages 8, 10, 16, 17, 25.
8. Look for and express regularity in repeated reasoning. Pages 9, 16, 17.

Standards for Mathematical Content—High School

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC.8-12.G.CO.1</td>
<td>Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. 1-1, 1-2, 1-4, 1-5</td>
</tr>
<tr>
<td>CC.9-12.G.SRT.7</td>
<td>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.* 1-3</td>
</tr>
<tr>
<td>CC.9-12.G.MG.1</td>
<td>Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* 1-6</td>
</tr>
</tbody>
</table>
In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

**Big Ideas**

1. Describing geometric figures
2. Measuring geometric figures
3. Understanding equality and congruence

**Key Vocabulary**

- undefined terms
- point, line, plane
- defined terms
- line segment, endpoints
- ray, opposite rays
- postulate, axiom
- congruent segments
- midpoint
- segment bisector
- acute, right, obtuse, straight angles
- congruent angles
- angle bisector
- linear pair
- vertical angles
- polygon
- convex, concave
- n-gon
- equilateral, equiangular, regular

**Why?**

Geometric figures can be used to represent real-world situations. For example, you can show a climber’s position along a stretched rope by a point on a line segment.

**Animated Geometry**

The animation illustrated below helps you answer a question from this chapter: How far must a climber descend to reach the bottom of a cliff?

[Image of the animation showing a climber and a cliff with a question and answer options]

[Link to animated geometry at my.hrw.com]
1.1 Identify Points, Lines, and Planes

**Warm-Up Exercises**

Also available online

Graph each inequality.

1. \(x \leq 1\)

2. \(-2 \leq x \leq 3\)

3. Juan has more than 5 but fewer than 11 fish in his aquarium.
   Write an inequality to express the number of fish Juan has.
   \(5 < f < 11\)

**Notetaking Guide**

Available online

Promotes interactive learning and notetaking skills.

**Pacing**

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with next lesson
*See Teaching Guide/Lesson Plan.*

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**Essential Question**

How do you name geometric figures? Tell students they will learn how to answer this question by learning about labeling points, lines, segments, rays, and planes.

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**Key Vocabulary**

- undefined terms
- point
- line
- plane
- collinear points
- coplanar points
- defined terms
- line segment
- endpoints
- ray
- opposite rays
- intersection

---

**In the diagram of a football field, the positions of players are represented by points.**

The yard lines suggest lines, and the flat surface of the playing field can be thought of as a plane.

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**In geometry, the words point, line, and plane are undefined terms.**

These words do not have formal definitions, but there is agreement about what they mean.

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**KEY CONCEPT**

**Undefined Terms**

**Point** A point has no dimension. It is represented by a dot.

**Line** A line has one dimension. It is represented by a line with two arrowheads, but it extends without end.

**Plane** A plane has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

**Collinear points** are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

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**For Your Notebook**

**C.C.9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Example 1  Name points, lines, and planes

Visual Reasoning
There is a line through points $S$ and $Q$ that is not shown in the diagram. Try to imagine what plane $SPQ$ would look like if it were shown.

a. Give two other names for $PQ$ and for plane $R$.
b. Name three points that are collinear. Name four points that are coplanar.

Solution
a. Other names for $PQ$ are $QP$ and line $n$. Other names for plane $R$ are plane $SVT$ and plane $PTV$.
b. Points $S$, $P$, and $T$ lie on the same line, so they are collinear. Points $S$, $P$, $T$, and $V$ lie in the same plane, so they are coplanar.

Guided Practice for Example 1
1. Use the diagram in Example 1. Give two other names for $ST$. Name a point that is not coplanar with points $Q$, $S$, and $T$.  

Sample answer: $TS$, $PT$, point $V$

Defined Terms In geometry, terms that can be described using known words such as point or line are called defined terms.

Key Concept Defined Terms: Segments and Rays

- **Line** $AB$ (written as $\overline{AB}$) and points $A$ and $B$ are used here to define the terms below.

- **Segment** The **line segment** $AB$, or **segment** $AB$, (written as $\overline{AB}$) consists of the **endpoints** $A$ and $B$ and all points on $\overline{AB}$ that are between $A$ and $B$.
  
  Note that $\overline{AB}$ can also be named $\overline{BA}$.

<table>
<thead>
<tr>
<th>Ray</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>ray</td>
<td>$\overline{AB}$</td>
</tr>
<tr>
<td>endpoint</td>
<td>$A$</td>
</tr>
<tr>
<td>$B$</td>
<td></td>
</tr>
</tbody>
</table>

Note that $\overline{AB}$ and $\overline{BA}$ are different rays.

If point $C$ lies on $\overline{AB}$ between $A$ and $B$, then $\overline{AC}$ and $\overline{CB}$ are **opposite rays**.

Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

Extra Example 1

a. Give two other names for $BD$.
b. Give another name for plane $T$.
c. Name three points that are collinear. $A$, $B$, $C$
d. Name four points that are coplanar. $A$, $B$, $C$, $E$

An Animated Geometry activity is available online for Example 1. This activity is also part of Power Presentations.
**Example 2** Name segments, rays, and opposite rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint $J$. Which of these rays are opposite rays?

**Solution**

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint $J$ are $\overrightarrow{JF}$, $\overrightarrow{JC}$, $\overrightarrow{JE}$, and $\overrightarrow{JJ}$. The pairs of opposite rays with endpoint $J$ are $\overrightarrow{JE}$ and $\overrightarrow{JJ}$, and $\overrightarrow{JC}$ and $\overrightarrow{JJ}$.

**Guided Practice** for Example 2

1. Use the diagram in Example 2.
2. Give another name for $\overrightarrow{EF}$, $\overrightarrow{EF}$
3. Are $\overrightarrow{HF}$ and $\overrightarrow{HF}$ the same ray? Are $\overrightarrow{HF}$ and $\overrightarrow{HG}$ the same ray? Explain.

No; the rays have different endpoints; yes; points $J$ and $G$ lie on the same side of $H$.

**Intersections** Two or more geometric figures intersect if they have one or more points in common. The intersection of the figures is the set of points the figures have in common. Some examples of intersections are shown below.

- The intersection of two different lines is a point.
- The intersection of two different planes is a line.

**Example 3** Sketch intersections of lines and planes

a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

c. Sketch a plane and a line that intersects the plane at a point.

**Solution**

a. 

b. 

 c. 

**Avoiding Common Errors**

- Can a line intersect a plane in only two points? Explain. No; a line can intersect a plane in one point if it does not lie in the plane or in an infinite number of points if it does lie in the plane, but never in only two points.

- Students may forget to include the line, segment, or ray symbol above letters. Remind them to be sure to include the correct symbol so they can determine whether the named figure is a line, segment, or ray.

**Mathematical Practice**

**Key Question**

Example 3

- Can a line intersect a plane in only two points? Explain. No; a line can intersect a plane in one point if it does not lie in the plane or in an infinite number of points if it does lie in the plane, but never in only two points.

**Kinesthetic Learners** While discussing Example 3, have students simulate the three cases by using a sheet of paper to represent the plane and a pen or pencil to represent the line. Then have students experiment to see that these are the only three possibilities involving a plane and a line. See also the Differentiated Instruction Resources for more strategies.
**Example 4** Sketch intersections of planes

Sketch two planes that intersect in a line.

**Solution**

**STEP 1** Draw a vertical plane. Shade the plane.

**STEP 2** Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

**STEP 3** Draw the line of intersection.

**Guided Practice** for Examples 3 and 4

1. Sketch two different lines that intersect a plane at the same point. See margin.

Use the diagram at the right.

2. Name the intersection of $\overline{FQ}$ and line $k$. Point $M$
3. Name the intersection of plane $A$ and plane $B$. Line $k$
4. Name the intersection of line $k$ and plane $A$. Line $k$

**1.1 Exercises**

**Skill Practice**

1. **Vocabulary** Write in words what each of the following symbols means.
   - $Q$ point $Q$
   - $\overline{MN}$ line segment $MN$
   - $\overline{ST}$ ray $ST$
   - $\overline{FG}$ line $FG$

2. **Writing** Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? Explain. See margin.

**Naming Points, Lines, and Planes** In Exercises 3–7, use the diagram.

3. Give two other names for $\overline{WQ}$. $\overline{QW}$, line $g$
4. Give another name for plane $V$. $\overline{WQ}$, plane $V$
5. Name three points that are collinear. Then name a fourth point that is not collinear with these three points. Sample answer: points $R, Q, S$, point $T$
6. Name a point that is not coplanar with $R, S$, and $T$. Point $W$
7. **Writing** Is point $W$ coplanar with points $Q$ and $R$? Explain. Yes; through any three points not on the same line, there is exactly one plane.

**Extra Example 4** Sketch two planes that do not intersect in a line.

**Mathematical Practice Example 4**

- **Key Question** Can two planes intersect in a segment? Explain. No; since the intersecting planes extend without end, their intersection must be a line.

**Closing the Lesson**

Have students summarize the major points of the lesson and answer the Essential Question: How do you name geometric figures?

- You can sketch and name points, lines, planes, segments, and rays.
- A line may intersect a plane in one point or lie in the plane.
- The intersection of two planes is a line.

Use one letter to name a point, two letters to name the endpoints of a segment, two letters to name the endpoint and one other point of a ray, and two letters to name any two points of a line. Use three letters for three noncollinear points in a plane to name the plane.

**Guided Practice**

4. Sample:

**Skill Practice**

2. Yes; no; collinear points are points that lie on the same line and therefore in the same plane, while coplanar points lie in the same plane but not necessarily on the same line.
4 PRACTICE AND APPLY

Assignment Guide
Answers for all exercises available online

Basic:
Day 1: SRH p. SR 8 Exs. 1–6
Exs. 1–16, 17–27 odd, 40–44
Average:
Day 1:
Exs. 1, 2, 3–11 odd, 12–16, 20–26, 27–37 odd, 40–45
Advanced:
Day 1:
Exs. 1, 5–7, 10, 11, 13–16, 20–38 even, 39–46*
Block:
Exs. 1, 2, 3–11 odd, 12–16, 20–26, 27–37 odd, 40–45 (with the next lesson)

Differentiated Instruction
See Differentiated Instruction Resources for suggestions on addressing the needs of a diverse classroom.

Homework Check
For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 8, 14, 21, 40
Average: 5, 8, 14, 20, 41
Advanced: 6, 13, 22, 26, 42

Extra Practice
• Student Edition
• Chapter Resource Book: Practice levels A, B, C

Practice Worksheet
An easily-readable reduced practice page can be found at the beginning of this chapter.

EXAMPLE 2
For Exs. 8–13

10. \( \overrightarrow{WX} \) and \( \overrightarrow{WV} \), \( \overrightarrow{VX} \) and \( \overrightarrow{VZ} \)

12. ERROR ANALYSIS A student says that \( \overrightarrow{WV} \) and \( \overrightarrow{VZ} \) are opposite rays because they have the same endpoint. Describe the error. Point \( V \) must lie between points \( W \) and \( Z \), which means the three points must be collinear.

13. ★ MULTIPLE CHOICE Which statement about the diagram is true? Choose one.

A. \( A, B, \) and \( C \) are collinear.

B. \( C, D, E, \) and \( G \) are coplanar.

C. \( B \) lies on \( \overrightarrow{GE} \).

D. \( \overrightarrow{EF} \) and \( \overrightarrow{FD} \) are opposite rays.

SKETCHING INTERSECTIONS Sketch the figure described. 14, 15. See margin.

14. Three lines that lie in a plane and intersect at one point

One line that lies in a plane, and one line that does not lie in the plane

15. ★ MULTIPLE CHOICE Line \( \overrightarrow{AB} \) and line \( \overrightarrow{CD} \) intersect at point \( E \), with point \( E \) between \( A \) and \( B \) and between \( C \) and \( D \). Which rays are opposite rays? A

A. \( \overrightarrow{BC} \) and \( \overrightarrow{DB} \)

B. \( \overrightarrow{CE} \) and \( \overrightarrow{ED} \)

C. \( \overrightarrow{AB} \) and \( \overrightarrow{BA} \)

D. \( \overrightarrow{AE} \) and \( \overrightarrow{EB} \)

READING DIAGRAMS In Exercises 17–22, use the diagram at the right.

17. Name the intersection of \( \overrightarrow{FK} \) and \( \overrightarrow{FR} \), point \( R \)

18. Name the intersection of plane \( EFG \) and plane \( FGS \).

19. Name the intersection of plane \( PQS \) and plane \( HGS \).

20. Are points \( P, Q, \) and \( F \) collinear? Are they coplanar?

21. Are points \( P \) and \( G \) collinear? Are they coplanar?

22. Name three planes that intersect at point \( E \).

Sample answer: plane \( PEF \), plane \( PEH \), plane \( HEF \)

23. SKETCHING PLANES Sketch plane \( J \) intersecting plane \( K \). Then draw a line \( \ell \) in plane \( J \) that intersects plane \( K \) at a single point. See margin.

24. NAMING RAYS Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.

25. SKETCHING Draw three noncollinear points \( I, K, \) and \( L \). Sketch \( JK \) and add a point \( M \) on \( JK \). Then sketch \( ML \). See margin.

26. SKETCHING Draw two points \( P \) and \( Q \). Then sketch \( \overrightarrow{PQ} \). Add a point \( R \) on the ray so that \( Q \) is between \( P \) and \( R \). See margin.
**ALGEBRA**  In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. \( y = x - 4; A(5, 1) \)  \( \text{on the line} \)
28. \( y = x + 1; A(-1, 0) \)  \( \text{not on the line} \)
29. \( y = 3x + 4; A(7, 1) \)  \( \text{not on the line} \)
30. \( y = 4x + 2; A(1, 6) \)  \( \text{on the line} \)
31. \( y = 3x - 2; A(-1, -5) \)  \( \text{on the line} \)
32. \( y = -2x + 8; A(-4, 0) \)  \( \text{not on the line} \)

**GRAPHING**  Graph the inequality on a number line. Tell whether the graph is a segment, a ray, or rays, a point, or a line. See margin for art.

33. \( x \leq 3 \) ray
34. \( x \geq -4 \) ray
35. \(-7 \leq x \leq 4 \) segment
36. \( x \geq 5 \) or \( x \leq -2 \) rays
37. \( x \geq -1 \) or \( x \leq 5 \) line
38. \(|x| \leq 0 \) point

39. **CHALLENGE**  Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch. See margin for art.

a. None of the three planes intersect. **possible**
b. The three planes intersect in one line. **possible**
c. The three planes intersect in one point. **possible**
d. Two planes do not intersect. The third plane intersects the other two. **possible**
e. Exactly two planes intersect. The third plane does not intersect the other two. **not possible**

**PROBLEM SOLVING**

40. \( \text{intersecting lines} \)
41. \( \text{intersection of a line and a plane} \)
42. \( \text{intersecting planes} \)

43. **SHORT RESPONSE**  Explain why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

44. **SURVEYING**  A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

a. When the tripod is sitting on a level surface, are the tips of the legs coplanar? **yes**
b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? **Explain.**
45. **MULTI-STEP PROBLEM** In a perspective drawing, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a vanishing point. The diagram shows a drawing of a house with two vanishing points. 45a–c. See margin.

![Perspective Drawing](image)

a. Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.

b. Label the intersection of $\overline{EV}$ and $\overline{AW}$ as C. Label the intersection of $\overline{FW}$ and $\overline{BW}$ as H.

c. Using heavy dashed lines, draw the hidden edges of the house: $\overline{AC}$, $\overline{EC}$, $\overline{BH}$, $\overline{FH}$, and $\overline{CH}$.

46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.

![Street Intersections](image)

a. A traffic light is needed at each intersection. How many traffic lights are needed if there are 2 streets in the town? 6 streets? **10 traffic lights; 15 traffic lights**

b. Describe a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town. Let $n$ represent the number of streets. To find the additional number of traffic lights needed, find $n - 1$. 
1.2 Use Segments and Congruence

You learned about points, lines, and planes. You will use segment postulates to identify congruent segments. So you can calculate flight distances, as in Ex. 33.

Key Vocabulary
- postulate, axiom
- coordinate
- distance
- between
- congruent segments

In Geometry, a rule that is accepted without proof is called a postulate or axiom. A rule that can be proved is called a theorem, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

**POSTULATE**

**POSTULATE 1 Ruler Postulate**

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the coordinate of the point.

The distance between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.

In the diagrams above, the small numbers in the coordinates x₁ and x₂ are called *subscripts*. The coordinates are read as “x sub one” and “x sub two.” The distance between points A and B, or AB, is also called the length of AB.

**EXAMPLE 1 Apply the Ruler Postulate**

Measure the length of ST to the nearest tenth of a centimeter.

\[ \text{Solution} \]

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, if you align S with 2, T appears to align with 5.4.

\[ \text{The length of } ST \text{ is about 3.4 centimeters.} \]

**Notetaking Guide**

Available online
Promotes interactive learning and notetaking skills.

**Pacing**
Basic: 1 day  
Average: 1 day  
Advanced: 1 day  
Block: 0.5 block with previous lesson

- See Teaching Guide/Lesson Plan.

**Focus and Motivate**

**Essential Question**

Big Idea 3
What are congruent segments? 
Tell students they will learn how to answer this question by finding lengths of segments.

**Standards for Mathematical Content High School**

CC.9-12.G.C0.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

CC.9-12.G.C0.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
**Motivating the Lesson**

Ask students to think of three places they go that are in a straight line, perhaps their house, a friend’s house, and school. Tell them that in this lesson they will learn how to find the third distance between these places when they are given the other two.

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**TEACH**

**Extra Example 1**

Measure the length of \( \overline{PQ} \) to the nearest tenth of a centimeter.

\[
3.2 \text{ cm}
\]

**Key Question Example 1**

- What if you had aligned \( S \) with \( 1 \) instead of \( 2 \)? The distance is still 3.4, but this time found by using \( |4.4 - 1| \).

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**Extra Example 2**

The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.

**MAPS** The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.

**Solution**

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

\[
LS = LT + TS = 380 + 360 = 740
\]

The distance from Lubbock to St. Louis is about 740 miles.

---

**Guided Practice** for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest \( \frac{1}{8} \) inch.

1. \( M \) to \( N \)

\[
\text{15 in.}
\]

2. \( P \) to \( Q \)

\[
\text{12 in.}
\]

3. Use the Segment Addition Postulate to find \( XZ \).

4. In the diagram, \( WY = 30 \). Can you use the Segment Addition Postulate to find the distance between points \( W \) and \( Z \)? Explain your reasoning. No; \( Y \) is not between \( W \) and \( Z \).

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**Differentiated Instruction**

**English Learners** Have students work problems similar to Example 2 using maps of other countries. In particular, find locations where English learners or their family members have come from.

See also the Differentiated Instruction Resources for more strategies.
**Example 3** Find a length

Use the diagram to find $GH$.

![Diagram showing points F, G, H, and segments FG, GH, FH, and 36]

**Solution**

Use the Segment Addition Postulate to write an equation. Then solve the equation to find $GH$.

- $FH = FG + GH$  
  Segment Addition Postulate
- $36 = 21 + GH$  
  Substitute 36 for $FH$ and 21 for $FG$.
- $15 = GH$  
  Subtract 21 from each side.

**Congruent Segments** Lines segments that have the same length are called congruent segments. In the diagram below, you can say “the length of $\overline{AB}$ is equal to the length of $\overline{CD}$,” or you can say “$\overline{AB}$ is congruent to $\overline{CD}$.” The symbol $\equiv$ means “is congruent to.”

![Diagram showing segments AB and CD with lengths AB = CD]

**Example 4** Compare segments for congruence

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(-1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

**Solution**

To find the length of a horizontal segment, find the absolute value of the difference of the $x$-coordinates of the endpoints.

- $JK = |2 - (-3)| = 5$  
  Use Ruler Postulate.

To find the length of a vertical segment, find the absolute value of the difference of the $y$-coordinates of the endpoints.

- $LM = |-2 - 3| = 5$  
  Use Ruler Postulate.

$\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \equiv \overline{LM}$.

**Guided Practice** for Examples 3 and 4

5. Use the diagram at the right to find $WX$.

6. Plot the points $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent. No

**Differentiated Instruction**

**Below Level** To stress the difference between equal and congruent, show students two identical one foot rulers. Ask students what the length of each ruler is and have them write the relationship 12 in. $\equiv$ 12 in. (or 1 ft $\equiv$ 1 ft). Turn the rulers over, label one as $A-B$ and the other as $C-D$. Tell students to write the relationship between $\overline{AB}$ and $\overline{CD}$. Students should write $\overline{AB} \equiv \overline{CD}$.

See also the Differentiated Instruction Resources for more strategies.
1.2 EXERCISES

**SKILL PRACTICE**

In Exercises 1 and 2, use the diagram at the right.

1. **VOCABULARY** Explain what \( MN \) means and what \( MN \) means.

   \( MN \) means segment \( MN \) while \( MN \) is the length of \( MN \).

2. **WRITING** Explain how you can find \( PN \) if you know \( PQ \) and \( QN \). How can you find \( PN \) if you know \( MP \) and \( MN \)? Find the sum \( PQ + QN \); find the difference \( MN - MP \).

**MEASUREMENT**

Measure the length of the segment to the nearest tenth of a centimeter.

3. \( A \) \( B \) \( 2.1 \text{ cm} \)
4. \( C \) \( D \) \( 3.2 \text{ cm} \)
5. \( E \) \( F \) \( 3.5 \text{ cm} \)

**SEGMENT ADDITION POSTULATE**

Find the indicated length.

6. Find \( MP \). \( 23 \)
7. Find \( RT \). \( 44 \)
8. Find \( UW \). \( 65 \)

9. Find \( XY \). \( 23 \)
10. Find \( BC \). \( 15 \)
11. Find \( DE \). \( 13 \)

12. **ERROR ANALYSIS** In the figure at the right, \( AC = 14 \) and \( AB = 9 \). Describe and correct the error made in finding \( BC \).

   \( 9 \) should be subtracted from \( 14 \), not added;
   \( BC = 14 - 9 = 5 \).

**CONGRUENCE**

In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

13. \( A(0, 1), B(4, 1), C(1, 2), D(1, 6) \); \( AB \) and \( CD \) congruent

14. \( J(-6, -8), K(-6, 2), L(-2, -4), M(-6, -4); JK \) and \( LM \) not congruent

15. \( R(-200, 300), S(200, 300), T(300, -200), U(300, 100); RS \) and \( TU \) not congruent

**ALGEBRA**

Use the number line to find the indicated distance.

16. \( JK \) \( 3 \)
17. \( JL \) \( 7 \)
18. \( JM \) \( 12 \)
19. \( KM \) \( 9 \)

20. **SHORT RESPONSE** Use the diagram. Is it possible to use the Segment Addition Postulate to show that \( FB > CB \) or that \( AC > DB \)? Explain.

   Yes, since \( FB = FC + CB \), then \( FB > CB \); no, the relationship between \( AD \) and \( BC \) is not known.
FINDING LENGTHS In the diagram, points V, W, X, Y, and Z are collinear, VZ = 52, XZ = 20, and WX = XY = YZ. Find the indicated length.

21. WX 10  
22. VW 22  
23. WY 20  
24. VX 32  
25. WZ 30  
26. VY 42

27. ★ MULTIPLE CHOICE Use the diagram.
   What is the length of \( \overline{DE} \)?
   (A) 1  
   (B) 4.4  
   (C) 10  
   (D) 16

28. \((2x + 10) + (x - 4) = 21; 5; 20, 1 \)
29. \((3x - 16) + (4x - 8) = 60; 12; 20, 40 \)
30. \((2x - 8) + (3x - 10) = 17; 7; 6, 11 \)

ALGEBRA Point S is between R and T on \( \overline{RT} \). Use the given information to write an equation in terms of \( x \). Solve the equation. Then find \( RS \) and \( ST \).

28. \( RS = 2x + 10 \)  
29. \( RS = 3x - 16 \)  
30. \( RS = 2x - 8 \)

\( ST = 4x - 4 \)  
\( ST = 4x - 8 \)  
\( ST = 3x - 10 \)

\( RT = 21 \)  
\( RT = 60 \)  
\( RT = 17 \)

31. CHALLENGE In the diagram, \( \overline{AB} \cong \overline{BC} \), \( \overline{AC} \cong \overline{CD} \), and \( \overline{AD} \) = 12. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain.

\( AC = 8 \), \( CD = 6 \), \( AB = 3 \), \( BC = 4 \), \( BD = 9 \), \( AD = 12 \); four of the six segment lengths are greater than 3.

PROBLEM SOLVING

32. SCIENCE The photograph shows an insect called a walking stick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest \( \frac{1}{4} \) inch. About how much longer is the walking stick’s abdomen than its thorax?

33. MODEL AIRPLANE In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane’s position at three different points during its flight.

A Leave Cape Spear, Newfoundland
B Approximate position after about 1 day
C Land at Mannin Bay, Ireland, after nearly 38 hours

a. Find the total distance the model airplane flew. 1883 mi
b. The model airplane’s flight lasted nearly 38 hours. Estimate the airplane’s average speed in miles per hour.  

1.2 Use Segments and Congruence
34. **SHORT RESPONSE** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

   a. Use the scale to find the length of the yellow bar for each year. What does the length represent?
   
   b. For each year, find the percent of games lost by the team.
   
   c. Explain how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown. Sample answer: The sum of the lengths of the bars for wins and losses represents the total number of games played.

35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let \( A \) represent the point where the rope is secured at the top of the cliff, let \( B \) represent the climber’s position, and let \( C \) represent the point where the rope is secured at the bottom of the cliff.

   a. **Model** Draw and label a line segment that represents the situation. See margin.

   b. **Calculate** If \( AC \) is 52 feet and \( AB \) is 31 feet, how much farther must the climber descend to reach the bottom of the cliff? \( 21 \) ft

36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart. See margin.

<table>
<thead>
<tr>
<th></th>
<th>City A</th>
<th>City B</th>
<th>City C</th>
<th>City D</th>
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<td>City A</td>
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<td>City D</td>
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</tr>
</tbody>
</table>
# 1.3 Use Midpoint and Distance Formulas

## Before
You found lengths of segments.

## Now
You will find lengths of segments in the coordinate plane.

## Why?
So you can find an unknown length, as in Example 1.

### Key Vocabulary
- midpoint
- segment bisector

### Activity Fold a Segment Bisector

**STEP 1**
Draw $AB$ on a piece of paper.

**STEP 2**
Fold the paper so that $B$ is on top of $A$.

**STEP 3**
Label point $M$. Compare $AM$, $MB$, and $AB$.

### Midpoints and Bisectors
The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector **bisects** a segment.

![Diagram of midpoint and bisector](image)

- $M$ is the midpoint of $AB$.
- So, $AM = MB$ and $AM = MB$.

- $CD$ is a segment bisector of $AB$.
- So, $AD = DB$ and $AD = DB$.

### Example 1 Find Segment Lengths

**Skateboard** In the skateboard design, $\overline{WW}$ bisects $\overline{XY}$ at point $T$, and $XT = 39.9$ cm. Find $XY$.

**Solution**
Point $T$ is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

\[
XY = XT + TY \quad \text{Segment Addition Postulate}
\]

\[
= 39.9 + 39.9 \quad \text{Substitute.}
\]

\[
= 79.8 \quad \text{Add.}
\]

---

### Standards for Mathematical Content High School

**CC.9-12.G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*
Example 2  Use algebra with segment lengths

**ALGEBRA**  Point \( M \) is the midpoint of \( VW \). Find the length of \( VM \).

**Solution**

1. **REVIEW ALGEBRA**  For help with solving equations, see SR7.

   **STEP 1**  Write and solve an equation. Use the fact that \( VM = MW \).

   \[
   VM = MW \\
   4x - 1 = 3x + 3
   \]

   Write equation.

   Subtract 3 from each side.

   Add 1 to each side.

   **STEP 2**  Evaluate the expression for \( VM \) when \( x = 4 \).

   \[
   VM = 4x - 1 = 4(4) - 1 = 15
   \]

   So, the length of \( VM \) is 15.

   **CHECK**  Because \( VM = MW \), the length of \( MW \) should be 15. If you evaluate the expression for \( MW \), you should find that \( MW = 15 \).

   \[
   MW = 3x + 3 = 3(4) + 3 = 15
   \]

Guided Practice for Examples 1 and 2

In Exercises 1 and 2, identify the segment bisector of \( PQ \). Then find \( PQ \).

1. \[
\overline{MN}; \quad 3 \frac{3}{4} \quad \text{line } f; \quad 11 \frac{2}{3}
\]

2. \[
\overline{MN}; \quad 3 \frac{3}{4} \quad \text{line } f; \quad 11 \frac{2}{3}
\]

**COORDINATE PLANE**  You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

**KEY CONCEPT**  The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the \( x \)-coordinates and of the \( y \)-coordinates of the endpoints.

If \( A(x_1, y_1) \) and \( B(x_2, y_2) \) are points in a coordinate plane, then the midpoint \( M \) of \( AB \) has coordinates

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

**For Your Notebook**

16  Chapter 1  Essentials of Geometry
Example 3  Use the Midpoint Formula

a. FIND MIDPOINT The endpoints of $RS$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint $M$.

\[ M \left( \frac{1 + 4}{2}, \frac{-3 + 2}{2} \right) = M \left( \frac{5}{2}, -\frac{1}{2} \right) \]

- The coordinates of the midpoint $M$ are $\left( \frac{5}{2}, -\frac{1}{2} \right)$.

b. FIND ENDPOINT The midpoint of $JK$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint $K$.

\[ \frac{x + 1}{2} = 2 \quad \frac{y + 4}{2} = 1 \]

\[ 1 + x = 4 \quad 4 + y = 2 \]

\[ x = 3 \quad y = -2 \]

- The coordinates of endpoint $K$ are $(3, -2)$.

Guided Practice for Example 3

3. The endpoints of $AB$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint $M$. $(4, 5)$

4. The midpoint of $VW$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint $V$. $(6, 8)$

Distance Formula The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

KEY CONCEPT

For Your Notebook

The Distance Formula

If $(x_1, y_1)$ and $(x_2, y_2)$ are points in a coordinate plane, then the distance between $A$ and $B$ is

\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Inclusion The subscripts sometimes cause students to make errors in substitution with the Midpoint Formula and the Distance Formula. Remind students that the $x$-coordinate in the midpoint formula is the average of the two $x$-values, and the $y$-coordinate of the midpoint formula is the average of the two $y$-values. Similarly, the square of the distance in the distance formula is just the square of the difference in the $x$-values plus the square of the difference in the $y$-values.

See also the Differentiated Instruction Resources for more strategies.
Extra Example 4
What is the approximate length of \( \overline{PQ} \) with endpoints \( P(2, 5) \) and \( Q(-4, 8) \)?

\[ \begin{align*}
A & \quad 3.61 \\
B & \quad 6.71 \\
C & \quad 9.0 \\
D & \quad 13.15
\end{align*} \]

Key Question
Example 4

- To use the Pythagorean Theorem instead of the distance formula, where would the right angle be located? \((2, -1)\) or \((4, 3)\)

Reading Strategy
The symbol \( \approx \) is introduced in Example 4. Ask students to differentiate among the symbols \( = \), \( \approx \), and \( \cong \).

Closing the Lesson
Have students summarize the major points of the lesson and answer the Essential Question: how do you find the distance and the midpoint between two points in the coordinate plane?

- The midpoint of a segment is the point that divides the segment into two congruent parts.
- The length of a segment in the coordinate plane is the distance between its endpoints.

To find the distance between the points \((x_1, y_1)\) and \((x_2, y_2)\), use the distance formula,

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

To find the midpoint, use the midpoint formula,

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

Example 4
Standardized Test Practice
What is the approximate length of \( \overline{RS} \) with endpoints \( R(2, 3) \) and \( S(4, -1) \)?

\[ \begin{align*}
A & \quad 1.4 \text{ units} \\
B & \quad 4.0 \text{ units} \\
C & \quad 4.5 \text{ units} \\
D & \quad 6 \text{ units}
\end{align*} \]

Solution
Use the Distance Formula. You may find it helpful to draw a diagram.

\[ RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2} \]

\[ = \sqrt{2^2 + (-4)^2} \]

\[ = \sqrt{4 + 16} \]

\[ = \sqrt{20} \]

\[ = 4.47 \]

The correct answer is C. \( \quad \boxed{\text{C}} \)

Guided Practice for Example 4
5. No. Sample answer: When squaring the differences in the coordinates, you get the same answer as long as you choose the \( x \) and \( y \) values from the same point.

6. What is the approximate length of \( \overline{AB} \), with endpoints \( A(-3, 2) \) and \( B(1, -4) \)?

\[ \begin{align*}
A & \quad 6.1 \text{ units} \\
B & \quad 7.2 \text{ units} \\
C & \quad 8.5 \text{ units} \\
D & \quad 10.0 \text{ units}
\end{align*} \]
1. **VOCABULARY** Copy and complete: To find the length of \( AB \), with endpoints \( A(-7, 5) \) and \( B(4, -6) \), you can use the __Distance Formula__. 

2. **WRITING** Explain what it means to bisect a segment. Why is it impossible to bisect a line? **Divide a segment into two congruent segments:** a line has infinite length.

**EXAMPLE 1** for Exs. 3–10

**FINDING LENGTHS** Line \( l \) bisects the segment. Find the indicated length.

3. Find \( RT \) if \( RS = 5 \frac{3}{5} \) in.

4. Find \( UW \) if \( VW = \frac{5}{8} \) in.

5. Find \( EG \) if \( EF = 13 \) cm.

6. Find \( BC \) if \( AC = 19 \) cm.

7. Find \( QR \) if \( PR = 9 \frac{2}{3} \) in.

8. Find \( LM \) if \( LN = 137 \) mm.

9. **SEGMENT BISECTOR** Line \( RS \) bisects \( PQ \) at point \( R \). Find \( RQ \) if \( PQ = 4 \frac{3}{4} \) inches. \( 2 \frac{3}{4} \) in.

10. **SEGMENT BISECTOR** Point \( T \) bisects \( UV \). Find \( UV \) if \( UT = 2 \frac{7}{8} \) inches. \( 5 \frac{3}{4} \) in.

**EXAMPLE 2** for Exs. 11–16

**ALGEBRA** In each diagram, \( M \) is the midpoint of the segment. Find the indicated length.

11. Find \( AM \). 10

12. Find \( EM \). 42

13. Find \( JM \). 1

14. Find \( PR \). 98

15. Find \( SU \). 70

16. Find \( XZ \). 146

**EXAMPLE 3** for Exs. 17–30

**FINDING MIDPOINTS** Find the coordinates of the midpoint of the segment with the given endpoints.

17. \( C(3, 5) \) and \( D(7, 5) \) \( (5, 5) \)

18. \( B(0, 4) \) and \( F(4, 3) \) \( (2, \frac{1}{2}) \)

19. \( G(-4, 4) \) and \( H(6, 4) \) \( (1, 4) \)

20. \( R(-7, -5) \) and \( K(-3, 7) \) \( (-5, 1) \)

21. \( P(-8, -7) \) and \( Q(11, 5) \) \( (-\frac{1}{2}, -\frac{1}{2}) \)

22. \( S(-3, 3) \) and \( T(-8, 6) \) \( (-\frac{1}{2}, \frac{1}{2}) \)

23. **WRITING** Develop a formula for finding the midpoint of a segment with endpoints \( A(0, 0) \) and \( B(m, n) \). Explain your thinking.

\( \left( \frac{m + n}{2}, \frac{m}{2} \right) \) when \( x_1 \) and \( y_1 \) are replaced by zero in the Midpoint Formula and \( x_1 \) and \( y_1 \) are replaced by \( m \) and \( n \) the result is \( \left( \frac{m + n}{2}, \frac{m}{2} \right) \).
24. **ERROR ANALYSIS** Describe the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.  
\[
\left( \frac{8 + 2}{2}, \frac{3 + (-1)}{2} \right) = (5, 1).
\]

**FININD ENDPOINTS** Use the given endpoint $R$ and midpoint $M$ of $RS$ to find the coordinates of the other endpoint $S$.

25. $R(3, 0), M(0, 5)$  
26. $R(5, 1), M(1, 4)$  
27. $R(8, -2), M(5, 3)$

28. $R(-7, 11), M(2, 1)$  
29. $R(4, -6), M(-7, 8)$  
30. $R(-4, -6), M(3, -4)$

**DISTANCE FORMULA** Find the length of the segment. Round to the nearest tenth of a unit.

31. $P(1, 2), Q(5, 7)$  
32. $R(1, -2), S(4, 5)$  
33. $T(-1, 2), U(-2, -1)$

34. ★ **MULTIPLE CHOICE** The endpoints of $MN$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $MN$?

- A. 1.4 units  
- B. 7.2 units  
- C. 13 units  
- D. 18.4 units

**NUMBER LINE** Find the length of the segment. Then find the coordinate of the midpoint of the segment.

35. $P, Q$  
36. $R, S$  
37. $T, U$

38. $R(-8, 4), S(-3, 6)$  
39. $T(3, 0), U(2, 4)$  
40. $V(-4, 6), W(-1, 2)$

41. ★ **MULTIPLE CHOICE** The endpoints of $EF$ are $L(-2, 2)$ and $P(3, 1)$. The endpoints of $GH$ are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?

- A. 2.24  
- B. 2.86  
- C. 5.10  
- D. 7.96

42. ★ **SHORT RESPONSE** One endpoint of $PQ$ is $P(-2, 4)$. The midpoint of $PQ$ is $M(1, 0)$. Explain how to find $PQ$.

**COMPARING LENGTHS** The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

43. $AB: A(0, 2), B(-3, 8)$  
44. $EF: E(1, 4), F(5, 1)$  
45. $GH: G(2, 2), H(0, -4)$

46. ★ **ALGEBRA** Points $S, T,$ and $P$ lie on a number line. Their coordinates are 0, 1, and $x$, respectively. Given $SP = PT$, what is the value of $x$?

- A. 1  
- B. 2  
- C. 3  
- D. 4

47. **CHALLENGE** $M$ is the midpoint of $JK$. $JM = \frac{x}{8}$ and $JK = \frac{3x}{4} - 6$. Find $MK$.
48. WINDMILL In the photograph of a windmill, $\overline{ST}$ bisects $\overline{QR}$ at point $M$. The length of $\overline{QM}$ is $18 \frac{1}{2}$ feet. Find $QR$ and $MR$. $QR = 37$ ft, $MR = 18 \frac{1}{2}$ ft

49. DISTANCES A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house? See margin for art; 2.85 km

ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.

50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.
   a. $A$ and $B$ 3.2 m  
   b. $B$ and $C$ 4.5 m  
   c. $C$ and $D$ 3.6 m  
   d. $A$ and $D$ 5 m  
   e. $B$ and $D$ 2.2 m  
   f. $A$ and $C$ 5.1 m

51. Which two objects are closest to each other? Which two are farthest apart?

52. WATER POLO The diagram shows the positions of three players during part of a water polo match. Player $A$ throws the ball to Player $B$, who then throws it to Player $C$. How far did Player $A$ throw the ball? How far did Player $B$ throw the ball? How far would Player $C$ throw the ball if he had thrown it directly to Player $B$? Round all answers to the nearest tenth of a meter. 10.4 m; 9.2 m; 18.3 m

1.3 Use Midpoint and Distance Formulas
5. **EXTENDED RESPONSE** As shown, a path goes around a triangular park.

a. Find the distance around the park to the nearest yard. **191 yd**

b. A new path and a bridge are constructed from point Q to the midpoint M of PR. Find QM to the nearest yard. **40 yd**

c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? Explain. About 1.5 min; find the total distance, about 230 yards, and divide by 150 yards per minute.

54. **CHALLENGE** \( \overline{AB} \) bisects \( \overline{CD} \) at point \( M \), \( \overline{CD} \) bisects \( \overline{AB} \) at point \( M \), and \( AB = 4 \cdot CM \). Describe the relationship between \( AM \) and \( CD \). They are equal lengths.

---

**QUIZ**

1. Sketch two lines that intersect the same plane at two different points.
   The lines intersect each other at a point not in the plane. See margin.

In the diagram of collinear points, \( AE = 26 \), \( AD = 15 \), and \( AB = BC = CD \). Find the indicated length.

2. \( DE \) **11**

3. \( AB \) **5**

4. \( AC \) **10**

5. \( BD \) **10**

6. \( CE \) **16**

7. \( BE \) **21**

8. The endpoints of \( \overline{RS} \) are \( R(-2, -1) \) and \( S(2, 3) \). Find the coordinates of the midpoint of \( \overline{RS} \). Then find the distance between \( R \) and \( S \). \((0, 1)\); about 5.7
1. **MULTI-STEP PROBLEM** The diagram shows existing roads (BD and DB) and a new road (CE) under construction.

   a. If you drive from point B to point E on existing roads, how far do you travel? 13 mi
   
   b. If you use the new road as you drive from B to E, about how far do you travel? Round to the nearest tenth of a mile if necessary. 11 mi
   
   c. About how much shorter is the trip from B to E if you use the new road? 2 mi

2. **GRIDDED ANSWER** Point M is the midpoint of PQ. If PM = 23x + 5 and MQ = 25x - 4, find the length of PQ. 217

3. **GRIDDED ANSWER** You are hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. You have been hiking for 45 minutes at an average speed of 2.4 kilometers per hour. How much farther (in kilometers) do you need to hike to reach the end of the trail? 3.6 km

4. **SHORT RESPONSE** The diagram below shows the frame for a wall. FH represents a vertical board, and FG represents a brace. If FG = 143 cm, does the brace bisect FH? If not, how long should FG be so that the brace does bisect FH? Explain. No; 1.4 m; half of 2.8 meters is 1.4 meters.

5. **SHORT RESPONSE** Point E is the midpoint of AB and the midpoint of CD. The endpoints of AB are A(-4, 5) and B(6, -5). The coordinates of point C are (2, 8). Find the coordinates of point D. Explain how you got your answer. See margin.

6. **OPEN-ENDED** The distance around a figure is its perimeter. Choose four points in a coordinate plane that can be connected to form a rectangle with a perimeter of 16 units. Then choose four other points and draw a different rectangle that has a perimeter of 16 units. Show how you determined that each rectangle has a perimeter of 16 units. See margin.

7. **SHORT RESPONSE** Use the diagram of a box. What are all the names that can be used to describe the plane that contains points B, F, and C? Name the intersection of planes ABC and BFE. Explain. See margin.

8. **EXTENDED RESPONSE** Jill is a salesperson who needs to visit towns A, B, and C. On the map below, AB = 18.7 km and BC = 2AB. Assume Jill travels along the road shown.

   a. Find the distance Jill travels if she starts at Town A, visits Towns B and C, and then returns to Town A. 112.2 km
   
   b. About how much time does Jill spend driving if her average driving speed is 70 kilometers per hour? about 1.6 h
   
   c. Jill needs to spend 2.5 hours in each town. Can she visit all three towns and return to Town A in an 8 hour workday? Explain. No; Sample answer: 3(2.5) + 1.6 > 8
1.4 Measure and Classify Angles

**Key Vocabulary**
- angle
- acute, right, obtuse, straight
- sides, vertex of an angle
- measure of an angle
- congruent angles
- angle bisector

**EXAMPLE 1 Name angles**

Name the three angles in the diagram.

- \( \angle WXZ \) or \( \angle XWZ \)
- \( \angle YXZ \) or \( \angle ZXY \)
- \( \angle WYX \) or \( \angle ZWX \)

You should not name any of these angles \( \angle X \) because all three angles have \( X \) as their vertex.

**MEASURING ANGLES**

A protractor can be used to approximate the measure of an angle. An angle is measured in units called degrees (\(^\circ\)). For instance, the measure of \( \angle WXZ \) in Example 1 above is 32\(^\circ\). You can write this statement in two ways.

**Words** The measure of \( \angle WXZ \) is 32\(^\circ\).

**Symbols** \( \angle WXZ = 32^\circ \)

**POSTULATE**

**POSTULATE 3 Protractor Postulate**

Consider \( \overrightarrow{OB} \) and a point \( A \) on one side of \( \overrightarrow{OB} \).

The rays of the form \( \overrightarrow{OA} \) can be matched one to one with the real numbers from 0 to 180.

The measure of \( \angle AOB \) is equal to the absolute value of the difference between the real numbers for \( \overrightarrow{OA} \) and \( \overrightarrow{OB} \).

**Common Core**

- **CC.9-12.G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- **CC.9-12.G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- **CC.9-12.G.CO.12** Make formal geometric constructions with a variety of tools and methods [compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.].
**CLASSIFYING ANGLES** Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

**Acute angle**
\[ 0 < \angle A < 90^\circ \]

**Right angle**
\[ \angle A = 90^\circ \]

**Obtuse angle**
\[ 90^\circ < \angle A < 180^\circ \]

**Straight angle**
\[ \angle A = 180^\circ \]

---

**EXAMPLE 2** **Measure and classify angles**

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. \( \angle KHJ \)  
   \[ \angle KHJ \]

b. \( \angle GHK \)  
   \[ \angle GHK \]

c. \( \angle GHJ \)  
   \[ \angle GHJ \]

d. \( \angle GHL \)  
   \[ \angle GHL \]

**Solution**

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

a. \( \overline{HH} \) is lined up with the \( 0^\circ \) on the inner scale of the protractor. \( \overline{HK} \) passes through \( 55^\circ \) on the inner scale. So, \( \angle KHJ = 55^\circ \). It is an acute angle.

b. \( \overline{HH} \) is lined up with the \( 0^\circ \) on the outer scale, and \( \overline{HK} \) passes through \( 125^\circ \) on the outer scale. So, \( \angle GHK = 125^\circ \). It is an obtuse angle.

c. \( \angle GHJ = 180^\circ \). It is a straight angle.

d. \( \angle GHL = 90^\circ \). It is a right angle.

---

**GUIDED PRACTICE** for Examples 1 and 2

1. Name all the angles in the diagram at the right. Which angle is a right angle? \( \angle POS, \angle RQS, \angle PQS \)

2. Draw a pair of opposite rays. What type of angle do the rays form? See margin for art; straight angle.

---

**POSTULATE 4** **Angle Addition Postulate**

**Words** If \( P \) is in the interior of \( \angle RST \), then the measure of \( \angle RST \) is equal to the sum of the measures of \( \angle RSP \) and \( \angle PST \).

**Symbols** If \( P \) is in the interior of \( \angle RST \), then \( \angle RST = \angle RSP + \angle PST \).

---

**Motivating the Lesson**

A piece of glass must be cut to fit the top of a table that has four sides of different lengths. Tell students that in this lesson they will learn how to use angle measures to ensure that the glass will fit perfectly.

---

**3 TEACH**

**Extra Example 1**

Name the three angles in the diagram. Sample answer: \( \angle MNO, \angle ONP, \angle MNP \).

**Extra Example 2**

Use the diagram to find the measure of each angle and classify the angle.

a. \( \angle DEC \)  
   \[ \angle DEC 90^\circ, \text{right} \]

b. \( \angle DEA \)  
   \[ \angle DEA 180^\circ, \text{straight} \]

c. \( \angle CEB \)  
   \[ \angle CEB 20^\circ, \text{acute} \]

d. \( \angle DEB \)  
   \[ \angle DEB 110^\circ, \text{obtuse} \]

---

**Differentiated Instruction**

**Inclusion** Students may have difficulty remembering which angles are acute and which are obtuse. Have students remember that “acute” means “sharp,” and relate this to the fact that the sides of an acute angle form a sharp point.

See also the **Differentiated Instruction Resources** for more strategies.
Study Strategy
Have students make up a phrase for remembering AROS, which lists acute, right, obtuse, and straight angles in order of size. One possibility is “All Rain Or Snow.”

Extra Example 3
If \( m \angle XYZ = 72^\circ \), find \( m \angle XYW \) and \( m \angle ZYW \). \( 51^\circ, 21^\circ \)

Key Question Example 3
• How can you check that the answer of \( 56^\circ \) and \( 89^\circ \) is correct? See if the angle measures add up to \( 145^\circ \).

Teaching Strategy
Tell students that we use the \( \equiv \) symbol to state that a pair of geometric figures have the same size and shape. It makes no sense to write \( 50^\circ \equiv 50^\circ \) or \( \frac{3}{2} \equiv \frac{3}{2} \) since measures and numbers are not geometric figures.

Example 3
Find angle measures

**ALGEBRA** Given that \( m \angle LKN = 145^\circ \), find \( m \angle LKM \) and \( m \angle MKN \).

**Solution**

**STEP 1** Write and solve an equation to find the value of \( x \).

\[
m \angle LKN = m \angle LKM + m \angle MKN
\]

\[
145^\circ = (2x \cdot 10)^\circ + (4x - 3)^\circ
\]

145 = 6x + 7

138 = 6x

\( x = 23 \)

**STEP 2** Evaluate the given expressions when \( x = 23 \).

\[
m \angle LKM = (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ
\]

\[
m \angle MKN = (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ
\]

\( \therefore m \angle LKM = 56^\circ \) and \( m \angle MKN = 89^\circ \).

Guided Practice for Example 3

Find the indicated angle measures.

3. Given that \( \angle KLM \) is a straight angle, find \( m \angle KLN \) and \( m \angle NLM \). \( 125^\circ, 55^\circ \)

4. Given that \( \angle EFG \) is a right angle, find \( m \angle BHG \) and \( m \angle HFG \). \( 60^\circ, 30^\circ \)

**CONGRUENT ANGLES** Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that “the measure of angle \( A \) is equal to the measure of angle \( B \),” or you can say “angle \( A \) is congruent to angle \( B \).”

---

**Differentiated Instruction**

Advanced Students have learned what it means for segments to be congruent and for angles to be congruent. Ask students to conjecture what it would mean for two triangles to be congruent, and then for two rectangles. Have students extend their conjectures to include any kind of polygon.

See also the Differentiated Instruction Resources for more strategies.
Example 4

Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DBG = 157^\circ$, what is $m\angle GKL$?

Solution

There are two pairs of congruent angles:

$\angle DBF \cong \angle JKL$ and $\angle DEG \cong \angle GKL$.

Because $\angle DEG \cong \angle GKL$, $m\angle DBG = m\angle GKL$. So, $m\angle GKL = 157^\circ$.

GUIDED PRACTICE for Example 4

Use the diagram shown at the right.

5. Identify all pairs of congruent angles in the diagram. $\angle T$ and $\angle S$, $\angle P$ and $\angle R$

6. In the diagram, $m\angle PQR = 130^\circ$, $m\angle QRS = 84^\circ$, and $m\angle TSR = 121^\circ$. Find the other angle measures in the diagram. $m\angle PTS = 121^\circ$, $m\angle OPT = 84^\circ$

Activity 4

Fold an Angle Bisector

1. Use a straightedge to draw and label an acute angle, $\angle ABC$.

2. Fold the paper so that $\overline{BC}$ is on top of $\overline{BA}$.

3. Draw a point $D$ on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

Extra Example 4

The figure shows angles formed by the legs of an ironing board. Identify the congruent angles. If $m\angle HGI = 40^\circ$, what is $m\angle GJK$?

$\angle HGI \cong \angle GJK$, $\angle GHK \cong \angle HKJ$, $40^\circ$

Key Question Example 4

• What is true about the measures of congruent angles? They are equal.

Activity Note

The purpose of this folding activity is to show students that an angle bisector divides an angle into two congruent angles. Ask students what the fold represents. The angle bisector.
An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on the previous page, \(BD\) bisects \(\angle ABC\). So, \(\angle ABD = \angle DBC\) and \(m\angle ABD = m\angle DBC\).

### Example 5
**Double an angle measure**

In the diagram at the right, \(YW\) bisects \(\angle XYZ\), and \(m\angle YXW = 18^\circ\). Find \(m\angle XYZ\).

**Solution**

By the Angle Addition Postulate, \(m\angle XYZ = m\angle YXW + m\angle WYZ\). Because \(YW\) bisects \(\angle XYZ\), you know that \(\angle YXW = \angle WYZ\).

So, \(m\angle YXW = m\angle WYZ\), and you can write

\[
m\angle XYZ = m\angle YXW + m\angle WYZ = 18^\circ + 18^\circ = 36^\circ.
\]

### Guided Practice for Example 5

7. Angle \(MNP\) is a straight angle, and \(NQ\) bisects \(\angle MNP\). Draw \(\angle MNP\) and \(\angle NQ\). Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles. **See margin for: 90°.**

### Exercises

#### Skill Practice

1. **Vocabulary** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight. **See margin.**

2. **Writing** Explain how to find the measure of \(\angle PQR\), shown at the right. The measure of \(\angle PQR\) is equal to the absolute value of the difference between the degree measures for \(\overline{QP}\) and \(\overline{QR}\).

#### Naming Angles and Angle Parts

In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.

3. \(\angle ABC, \angle BAC, \angle CBA; B, \overline{BA}, \overline{BC}\)

4. \(\angle NOQ, \angle Q, \angle TON; O, \overline{ON}, \overline{OT}\)

5. \(\angle MTP, \angle T, \angle PTM; T, \overline{TM}, \overline{TP}\)
6. **NAMING ANGLES** Name three different angles in the diagram at the right. \( \angle QRT, \angle QRS, \angle SRT \)

**CLASSIFYING ANGLES** Classify the angle with the given measure as acute, obtuse, right, or straight.

7. \( m\angle W = 180^\circ \) \hspace{1cm} 8. \( m\angle X = 30^\circ \) \hspace{1cm} 9. \( m\angle Y = 90^\circ \) \hspace{1cm} 10. \( m\angle Z = 95^\circ \)

**MEASURING ANGLES** Trace the diagram and extend the rays.

Use a protractor to find the measure of the given angle. Then classify the angle as acute, obtuse, right, or straight.

11. \( \angle JFL \) \( 90^\circ \); right \hspace{1cm} 12. \( \angle GFL \) \( 60^\circ \); acute \hspace{1cm} 13. \( \angle GFK \) \( 135^\circ \); obtuse \hspace{1cm} 14. \( \angle JFL \) \( 180^\circ \); straight

**NAMING AND CLASSIFYING** Give another name for the angle in the diagram below. Tell whether the angle appears to be acute, obtuse, right, or straight.

15. \( \angle ACB \) \hspace{1cm} 16. \( \angle ABC \) \hspace{1cm} 17. \( \angle BDF \) \hspace{1cm} 18. \( \angle ABC \) \hspace{1cm} 19. \( \angle BDC \) \hspace{1cm} 20. \( \angle BDC \)

**ANGLE ADDITION POSTULATE** Find the indicated angle measure.

21. \( \angle ABC \) \hspace{1cm} (a) \( \angle ACB \) \hspace{1cm} (b) \( \angle ACD \) \hspace{1cm} (c) \( \angle BCD \) \hspace{1cm} (d) \( \angle C \)

22. \( m\angle QST = \frac{90^\circ}{?} \) \hspace{1cm} 23. \( m\angle ADC = \frac{65^\circ}{?} \) \hspace{1cm} 24. \( m\angle NPM = \frac{101^\circ}{?} \)

**ALGEBRA** Use the given information to find the indicated angle measure.

25. Given \( m\angle WXYZ = 80^\circ \), find \( m\angle YXZ \). \hspace{1cm} 26. Given \( m\angle FJH = 168^\circ \), find \( m\angle FJG \).

27. **MULTIPLE CHOICE** In the diagram, the measure of \( \angle XYZ \) is \( 140^\circ \). What is the value of \( x? \)

\[
\begin{array}{ll}
\text{(A) } 27 & \text{(B) } 33 \\
\text{(C) } 67 & \text{(D) } 73 \\
\end{array}
\]
28. **CONGRUENT ANGLES** In the photograph below, \( m\angle AED = 34^\circ \) and \( m\angle ABD = 112^\circ \). Identify the congruent angles in the diagram. Then find \( m\angle BDA \) and \( m\angle ADB \).

![Diagram of angles](image)

**ANGLE BISECTORS** Given that \( \overline{WZ} \) bisects \( \angle XWY \), find the two angle measures not given in the diagram.

29. \( m\angle XWY = 104^\circ \), \( m\angle ZWY = 52^\circ \), \( m\angle XWZ = 136^\circ \), \( m\angle WXZ = 68^\circ \), \( m\angle XWZ = 35.5^\circ \), \( m\angle YWZ = 35.5^\circ \)

30. \( m\angle XWY = 104^\circ \), \( m\angle ZWY = 52^\circ \), \( m\angle XWZ = 136^\circ \), \( m\angle WXZ = 68^\circ \), \( m\angle XWZ = 35.5^\circ \), \( m\angle YWZ = 35.5^\circ \)

31. **ERROR ANALYSIS** \( \overline{KM} \) bisects \( \angle JKL \) and \( m\angle JKM = 30^\circ \). Describe and correct the error made in stating that \( m\angle JKL = 15^\circ \). Draw a sketch to support your answer. To find \( m\angle JKL \), \( m\angle JKM \) should be doubled, not halved; \( m\angle JKL = 60^\circ \); see margin for art.

**FINDING ANGLE MEASURES** Find the indicated angle measure.

32. \( a = 38^\circ \)

33. \( b = 38^\circ \)

34. \( c = 142^\circ \)

35. \( d = 37^\circ \)

36. \( e = 53^\circ \)

37. \( f = 37^\circ \)

38. **ERROR ANALYSIS** A student states that \( \overline{AD} \) can bisect \( \angle AGC \). Describe and correct the student’s error. Draw a sketch to support your answer.

**ALGEBRA** In each diagram, \( \overline{BD} \) bisects \( \angle ABC \). Find \( m\angle ABC \).

39. \( 156^\circ \)

40. \( 80^\circ \)

41. \( 134^\circ \)

42. \( 141^\circ \)

43. **SHORT RESPONSE** You are measuring \( \angle PQR \) with a protractor. When you line up \( \overline{QR} \) with the 20° mark, \( \overline{QP} \) lines up with the 80° mark. Then you move the protractor so that \( \overline{QR} \) lines up with the 15° mark. What mark does \( \overline{QP} \) line up with? Explain. 75°; both angle measures are 5° less.

44. **ALGEBRA** Plot the points in a coordinate plane and draw \( \angle ABC \). Classify the angle. Then give the coordinates of a point that lies in the interior of the angle. 44–47. See margin for art.

44. \( A(3, 3), B(0, 0), C(3, 0) \)

45. \( A(-5, 4), B(1, 4), C(-2, -2) \)

46. \( A(-5, 2), B(-2, -2), C(4, -3) \)

47. \( A(-3, -1), B(2, 1), C(6, -2) \)

48. \( A(-3, 0), B(2, 1), C(6, -2) \)

**See WORKED-OUT SOLUTIONS in Student Resources**
48. **ALGEBRA** Let \( (2x - 12)^\circ \) represent the measure of an acute angle. What are the possible values of \( x \)? 
\( 6 < x < 51 \)

49. **CHALLENGE** 
\( \overline{SV} \) bisects \( \angle RST \), \( \overline{SP} \) bisects \( \angle RSQ \), and \( \overline{SV} \) bisects \( \angle RSP \). The measure of \( \angle VSP \) is 17°. Find \( m \angle TSQ \). Explain.

50. **FINDING MEASURES** In the diagram, \( m \angle AEB = \frac{1}{2} \cdot m \angle CED \), and \( \angle AED \) is a straight angle. Find \( m \angle AEB \) and \( m \angle CED \). 
\( 30^\circ, 60^\circ \)

---

**Problem Solving**

51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of \( \angle LMN \) is 78° and the measure of \( \angle PMN \) is 47°. What is the measure of \( \angle LMP \)? 32°

52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road. 75°

---

**Avoiding Common Errors**

**Exercise 48** Students may write 2\( x - 12 \leq 90 \) but forget 2\( x - 12 > 0 \). Remind them that they need both inequalities to comply with the definition of an acute angle.

**Mathematical Reasoning**

**Exercise 50** Have students use \( x \) to represent \( m \angle CED \). Then they can write an equation to find \( x \).

**Reading Strategy**

**Exercise 53** Tell the students that reading and understanding a geometry exercise often requires reading not only the words but also a diagram. The diagram for this exercise contains several segments that are not used in answering the exercise. Students may want to make a separate sketch to show only the parts of the truss that are needed for coming up with the answer.

---

**Constructions**

In Exercises 53–55, use the photograph of a roof truss.

53. In the roof truss, \( \overrightarrow{BC} \) bisects \( \angle ABC \) and \( \angle DEF \), \( m \angle ABC = 112^\circ \), and \( \angle ABC = \angle DEF \). Find the measure of the following angles.
   a. \( m \angle DEF \)
   b. \( m \angle ABG \)
   c. \( m \angle CBG \)
   d. \( m \angle DEG \)

54. In the roof truss, \( \overrightarrow{GB} \) bisects \( \angle DGF \). Find \( m \angle DGE \) and \( m \angle FGE \).

55. Name an example of each of the following types of angles: acute, obtuse, right, and straight.
**GEOGRAPHY** For the given location on the map, estimate the measure of ∠PSL, where P is on the Prime Meridian (0° longitude), S is the South Pole, and L is the location of the indicated research station.

56. Macquarie Island about 156°
57. Dumont d’Urville about 140°
58. McMurdo about 167°
59. Mawson about 62°
60. Syowa about 33°
61. Vostok about 107°

---

62a. ∠AFB, ∠BFC, ∠CFD, ∠DFE, ∠AFD, ∠BFE, ∠AFC, ∠CFE

62b. ∠AFE is a straight angle and FC bisects ∠AFE and ∠BFD.

a. Which angles are acute? obtuse? right?

b. Identify the congruent angles.

c. If m∠AFB = 26°, find m∠DFE, m∠BFC, m∠CFD, m∠AFC, m∠AFD, and m∠BFD. Explain. See margin.

---

63. **CHALLENGE** Create a set of data that could be represented by the circle graph at the right. Explain your reasoning. Sample answer: In your pocket you have 4 pennies, 4 nickels, 4 dimes, 6 quarters, 6 dollar bills; nickels, dimes, pennies each represent \( \frac{1}{6} \), and quarters and dollar bills each represent \( \frac{1}{4} \).
Copy and Bisect Segments and Angles

**MATERIALS** • compass • straightedge

**QUESTION** How can you copy and bisect segments and angles?

A **construction** is a geometric drawing that uses a limited set of tools, usually a compass and straightedge. You can use a compass and straightedge (a ruler without marks) to construct a segment that is congruent to a given segment, and an angle that is congruent to a given angle.

**EXPLORE 1** Copy a segment

Use the following steps to construct a segment that is congruent to \( \overline{AB} \).

**STEP 1**

![Diagram](image1)

**STEP 2**

![Diagram](image2)

**STEP 3**

![Diagram](image3)

**EXPLORE 2** Bisect a segment

Use the following steps to construct a bisector of \( \overline{AB} \) and to find the midpoint \( M \) of \( \overline{AB} \). This construction is justified in the lesson **Use Perpendicular Bisectors**, Exercise 27.

**STEP 1**

![Diagram](image4)

**STEP 2**

![Diagram](image5)

**STEP 3**

![Diagram](image6)

**Mathematical Practices**

Use appropriate tools strategically.

1. **PLAN AND PREPARE**

**Explore the Concept**

- Students will copy and bisect a segment and an angle.
- This activity supplements the study of congruent segments and angles, and segment and angle bisectors.

**Materials**

Each student will need:

- compass
- straightedge

**Recommended Time**

Work activity: 15 min
Discuss results: 5 min

**Grouping**

Students should work individually.

2. **TEACH**

**Tips for Success**

In Explore 2, point out to students that it is important not to change the compass setting after they draw the first arc.

In Explore 4, students should note that the figure for Step 2 used a different compass setting from that used in Step 1. The same compass setting for all three arcs will also work.

**Alternative Strategy**

Demonstrate these constructions on the chalkboard or with a figure on the overhead projector.

**Key Discovery**

There are infinitely many bisectors of a segment but the one that is constructed in this activity is perpendicular to the segment. There is only one bisector of an angle.

**Common Core**

CC.9-12.G.CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).
**Explore 3** Copy an angle

Use the following steps to construct an angle that is congruent to \( \angle A \). In this construction, the **radius** of an arc is the distance from the point where the compass point rests (the **center** of the arc) to a point on the arc drawn by the compass. This construction is justified in the lesson **Use Congruent Triangles, Example 4**.

**Step 1**
Draw a segment
Draw a segment. Label a point \( D \) on the segment.

**Step 2**
Draw arcs
Draw an arc with center \( A \). Using the same radius, draw an arc with center \( D \).

**Step 3**
Draw arcs
Label \( B \), \( C \), and \( E \). Draw an arc with radius \( BC \) and center \( E \). Label the intersection \( F \).

**Step 4**
Draw a ray
Draw \( DF \).
\( \angle EDF = \angle BAC \).

**Explore 4** Bisect an angle

Use the following steps to construct an angle bisector of \( \angle A \). This construction is justified in the lesson **Use Congruent Triangles, Exercise 32**.

**Step 1**
Draw an arc
Place the compass at \( A \). Draw an arc that intersects both sides of the angle. Label the intersections \( C \) and \( B \).

**Step 2**
Draw arcs
Place the compass at \( C \). Draw an arc. Then place the compass point at \( B \). Using the same radius, draw another arc.

**Step 3**
Draw a ray
Label the intersection \( G \). Use a straightedge to draw a ray through \( A \) and \( G \). \( \overrightarrow{AG} \) bisects \( \angle A \).

**Draw Conclusions**
Use your observations to complete these exercises

1. **Describe** how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment.
   **Sample answer:** Copy the shorter line segment twice end-to-end.

2. Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge. **See margin.**
1.5 Describe Angle Pair Relationships

You used angle postulates to measure and classify angles.
You will use special angle relationships to find angle measures.
So you can find measures in a building, as in Ex. 53.

Key Vocabulary
- **complementary angles**
- **supplementary angles**
- **adjacent angles**
- **linear pair**
- **vertical angles**

Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the complement of the other. Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the supplement of the other.

Complementary angles and supplementary angles can be **adjacent angles** or **nonadjacent angles**. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.

**Complementary angles**
\[
\begin{align*}
\angle 1 & \quad \angle 2 \\
\angle 3 & \quad \angle 4 \\
\angle 5 & \quad \angle 6 \\
\angle 7 & \quad \angle 8
\end{align*}
\]

**Supplementary angles**

**EXAMPLE 1** Identify complements and supplements

**In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.**

**Solution**
Because 32° + 58° = 90°, \(\angle BAC\) and \(\angle RST\) are complementary angles.

Because 122° + 58° = 180°, \(\angle CAD\) and \(\angle RST\) are supplementary angles.

Because \(\angle BAC\) and \(\angle CAD\) share a common vertex and side, they are adjacent.

**AVOID ERRORS**
In Example 1, \(\angle DAC\) and \(\angle DAB\) share a common vertex. But they share common interior points, so they are not adjacent angles.

**GUIDED PRACTICE** for Example 1

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles. \(\angle FKG\) and \(\angle GKL\), \(\angle HKG\) and \(\angle GKL\), \(\angle FKG\) and \(\angle HKG\).

2. Are \(\angle KGH\) and \(\angle LGK\) adjacent angles? Are \(\angle FKG\) and \(\angle FGH\) adjacent angles? Explain.

**NOTETAKING GUIDE**
Available online
Promotes interactive learning and notetaking skills.

**PACING**
Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with next lesson
- See Teaching Guide/Lesson Plan.

**FOCUS AND MOTIVATE**

**Essential Question**

**Big Idea 2**
How do you identify complementary and supplementary angles? Tell students they will learn how to answer this question by finding the sum of the measures of two given angles.

**STANDARDS FOR MATHEMATICAL CONTENT**

**High School**

CC.9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

CC.9-12.G.CO.9 Prove theorems about lines and angles.
Motivating the Lesson
Parts for furniture that you assemble yourself often include parts that must fit together to form right angles or straight edges. Tell students that in this lesson they will learn terms that are useful in understanding how the angles for such parts are related.

3 TEACH

Example 2
Find measures of a complement and a supplement

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68^\circ$, find $m\angle 2$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56^\circ$, find $m\angle 3$.

Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 68^\circ = 22^\circ$$

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 56^\circ = 124^\circ$$

Example 3
Find angle measures

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCR$ and $m\angle ECD$.

Solution

**STEP 1** Use the fact that the sum of the measures of supplementary angles is $180^\circ$.

$$m\angle BCE + m\angle ECD = 180^\circ$$

**Write equation.**

$$\left(4x + 8\right)^\circ + \left(x + 2\right)^\circ = 180^\circ$$

**Substitute.**

$$5x + 10 = 180$$

**Combine like terms.**

$$5x = 170$$

**Subtract 10 from each side.**

$$x = 34$$

**Divide each side by 5.**

**STEP 2** Evaluate the original expressions when $x = 34$.

$$m\angle BCR = \left(4x + 8\right)^\circ = \left(4 \cdot 34 + 8\right)^\circ = 144^\circ$$

$$m\angle ECD = \left(x + 2\right)^\circ = \left(34 + 2\right)^\circ = 36^\circ$$

The angle measures are $144^\circ$ and $36^\circ$.

Guided Practice for Examples 2 and 3

3. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 82^\circ$, find $m\angle 1$. 18°

4. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117^\circ$, find $m\angle 4$. 63°

5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$. 26°, 64°

Inclusion

Students may confuse the definitions of complementary angles and supplementary angles. One memory device is to notice that both of the sets (complementary, supplementary) and (90°, 180°) are in alphabetical order and ascending order, respectively.

See also the Differentiated Instruction Resources for more strategies.
**ANGLE PAIRS** Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.

![Diagram of linear and vertical angles]

**Example 4** Identify angle pairs

Identify all of the linear pairs and all of the vertical angles in the figure at the right.

**Solution**

To find vertical angles, look for angles formed by intersecting lines.

- $\angle 1$ and $\angle 5$ are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

- $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.

**Example 5** Find angle measures in a linear pair

**ALGEBRA** Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.

**Solution**

Let $x^\circ$ be the measure of one angle. The measure of the other angle is $5x^\circ$. Then use the fact that the angles of a linear pair are supplementary to write an equation.

\[ x^\circ + 5x^\circ = 180^\circ \]

- Write an equation.

\[ 6x = 180 \]

- Combine like terms.

\[ x = 30 \]

- Divide each side by 6.

The measures of the angles are $30^\circ$ and $5(30^\circ) = 150^\circ$.

**Guided Practice** for Examples 4 and 5

6. Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? Explain.

7. The measure of an angle is twice the measure of its complement. Find the measure of each angle. $60^\circ, 30^\circ$

---

**Extra Example 4**

Identify all of the linear pairs and all of the vertical angles in the figure.

$\angle 2$ and $\angle 3$, $\angle 1$ and $\angle 2$; $\angle 1$ and $\angle 3$

**Key Question Example 4**

- The noncommon sides of $\angle 2$ and $\angle 3$ form an angle. What kind of angle pair will this angle form with $\angle 4$? **Vertical**

**Extra Example 5**

Two angles form a linear pair. The measure of one angle is 3 times the measure of the other angle. Find the measure of each angle. $45^\circ$, $135^\circ$

**Key Question Example 5**

- How would your equation change if one angle was $5^\circ$ more than the other? **It would be $x^\circ + (x + 5)^\circ = 180^\circ$**

**Closing the Lesson**

Have students summarize the major points of the lesson and answer the Essential Question: How do you identify complementary and supplementary angles?

- Complementary angles have a sum of $90^\circ$.
- Supplementary angles have a sum of $180^\circ$.
- Adjacent angles share a vertex but no common interior points, while a linear pair are adjacent angles whose noncommon sides are opposite rays.
- Vertical angles are angles whose sides form two pairs of opposite rays.

Add the measures of the angles. If the sum is $90^\circ$, the angles are complementary. If the sum is $180^\circ$, the angles are supplementary.
**CONCEPT SUMMARY**

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you *can conclude* from the diagram at the right:

- All points shown are coplanar.
- Points $A$, $B$, and $C$ are collinear, and $B$ is between $A$ and $C$.
- $\overline{AC}$, $\overline{BD}$, and $\overline{BE}$ intersect at point $B$.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point $E$ lies in the interior of $\angle DBC$.

In the diagram above, you *cannot conclude* that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.

---

**1.5 EXERCISES**

**SKILL PRACTICE**

1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain.* See margin.

2. **WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain.* See margin.

**IDENTIFYING ANGLES** Tell whether the indicated angles are adjacent.

3. $\angle ABD$ and $\angle DBC$ adjacent

4. $\angle WXY$ and $\angle XYZ$ not adjacent

5. $\angle LQM$ and $\angle NQM$ adjacent

**IDENTIFYING ANGLES** Name a pair of complementary angles and a pair of supplementary angles.

6. $\angle RTS$ and $\angle UWW$, $\angle JTS$ and $\angle UWV$

7. $\angle GLH$ and $\angle HJL$, $\angle GLJ$ and $\angle JLK$

---

No. Sample answer: Any two angles whose angle measures add up to $90^\circ$ are complementary, but they do not have to have a common vertex and side.

2. Yes; no. Sample answer: To be a linear pair, the noncommon sides of two adjacent angles must be opposite rays, which make a straight angle; supplementary angles need not be linear pairs because they can be nonadjacent.
**Example 2**
for Exs. 8–16

1. **Complementary Angles** \( \angle 1 \) and \( \angle 2 \) are complementary angles. Given the measure of \( \angle 1 \), find \( m \angle 2 \).

8. \( m \angle 1 = 43^\circ \), find \( m \angle 2 \).

9. \( m \angle 1 = 21^\circ \), find \( m \angle 2 \).

10. \( m \angle 1 = 89^\circ \), find \( m \angle 2 \).

11. \( m \angle 1 = 5^\circ \), find \( m \angle 2 \).

**Example 3**
for Exs. 17–19

1. **Supplementary Angles** \( \angle 1 \) and \( \angle 2 \) are supplementary angles. Given the measure of \( \angle 1 \), find \( m \angle 2 \).

12. \( m \angle 1 = 60^\circ \), find \( m \angle 2 \).

13. \( m \angle 1 = 155^\circ \), find \( m \angle 2 \).

14. \( m \angle 1 = 130^\circ \), find \( m \angle 2 \).

15. \( m \angle 1 = 27^\circ \), find \( m \angle 2 \).

16. **Multiple Choice** The arm of a crossing gate moves \( 37^\circ \) from vertical. How many degrees does the arm have to move so that it is horizontal?

- (A) \( 37^\circ \)
- (B) \( 53^\circ \)
- (C) \( 90^\circ \)
- (D) \( 143^\circ \)

**Example 4**
for Exs. 20–27

1. **Algebra** Find \( m \angle DEG \) and \( m \angle GEF \).

17. \( 135^\circ, 45^\circ \)

18. \( 67^\circ, 113^\circ \)

19. \( 54^\circ, 36^\circ \)

**Example 5**
for Exs. 28–30

1. **Identifying Angle Pairs** Use the diagram below. Tell whether the angles are **vertical angles**, a **linear pair**, or neither.

20. \( \angle 1 \) and \( \angle 4 \)

21. \( \angle 1 \) and \( \angle 2 \)

22. \( \angle 3 \) and \( \angle 5 \)

23. \( \angle 2 \) and \( \angle 3 \)

24. \( \angle 7 \), \( \angle 8 \), and \( \angle 9 \)

25. \( \angle 5 \) and \( \angle 6 \)

26. \( \angle 6 \) and \( \angle 7 \)

**Example 6**
for Exs. 31–34

1. **Error Analysis** Describe and correct the error made in finding the value of \( x \).

29. \( x^2 + 3x = 180^\circ \)

30. \( 4x = 180^\circ \)

31. \( x = 45 \)

**Example 7**
for Exs. 31–34

1. **Multiple Choice** The measure of one angle is \( 24^\circ \) greater than the measure of its complement. What are the measures of the angles?

- (A) \( 24^\circ \) and \( 66^\circ \)
- (B) \( 24^\circ \) and \( 156^\circ \)
- (C) \( 33^\circ \) and \( 57^\circ \)
- (D) \( 78^\circ \) and \( 102^\circ \)

**Example 8**
for Exs. 31–34

1. **Algebra** Find the values of \( x \) and \( y \).

31. \( 2y = 7x \)

32. \( 9y + 38 \)

33. \( 2y = (4x - 100)\)
Mathematical Reasoning
Exercises 34 and 35  Have students compare the wording in these two exercises. Have them describe in their own words how the answer to one of these exercises implies the answer to the other.

Exercises 46–48  Ask students to explain how they arrived at their answers. Some students may say that they simply looked at the clocks to decide. Ask how they would respond if someone challenged one of their answers.

INTERNET REFERENCE
Exercises 49–52  More information about the Rock and Roll Hall of Fame can be found at www.rockhall.com/museum

34. Never; an obtuse angle is larger than 90°, and it is not possible to have a complement of an angle that is greater than 90°.
35. Never; a straight angle is 180°, and it is not possible to have a complement of an angle that is 180°.
36. Sometimes; a straight angle does not have a supplement.
37. Always; the sum of complementary angles is 90°, so each angle must be less than 90°, making them acute.
38. Always; an acute angle is less than 90°, since the sum of supplementary angles is 180° and one of the angles is less than 90°, the other angle must be larger than 90°, which makes it obtuse.
45. \((x + 90)°\). Sample answer: Since \(m \angle GHJ = x°\) and \(\angle RST\) is its complement, \(m \angle RST = (90 - x)°\). \(\angle ABC\) is a supplement of \(\angle RST\) so \(m \angle ABC = 180° - (90 - x)°, m \angle ABC = (x + 90)°\).

REASONING  Tell whether the statement is always, sometimes, or never true.
Explain your reasoning. 34–38. See margin.
34. An obtuse angle has a complement.
35. A straight angle has a complement.
36. An angle has a supplement.
37. The complement of an acute angle is an acute angle.
38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES  \(\angle A\) and \(\angle B\) are complementary. Find \(m \angle A\) and \(m \angle B\).
39. \(m \angle A = (3x + 2)°\)  \(m \angle B = (x - 4)°\)  \(71°, 19°\)
40. \(m \angle A = (15x + 3)°\)  \(m \angle B = (5x - 13)°\)  \(78°, 12°\)
41. \(m \angle A = (11x + 24)°\)  \(m \angle B = (x + 18)°\)  \(68°, 22°\)

FINDING ANGLES  \(\angle A\) and \(\angle B\) are supplementary. Find \(m \angle A\) and \(m \angle B\).
42. \(m \angle A = (8x + 100)°\)  \(m \angle B = (2x + 50)°\)  \(124°, 56°\)
43. \(m \angle A = (2x - 20)°\)  \(m \angle B = (3x + 5)°\)  \(58°, 122°\)
44. \(m \angle A = (6x + 72)°\)  \(m \angle B = (2x + 28)°\)  \(132°, 48°\)

45. CHALLENGE  You are given that \(\angle GHJ\) is a complement of \(\angle RST\) and \(\angle RST\) is a supplement of \(\angle ABC\). Let \(m \angle GHJ = x°\). What is the measure of \(\angle ABC\)? Explain your reasoning. See margin.

PROBLEM SOLVING
A  IDENTIFYING ANGLES  Tell whether the two angles shown are complementary, supplementary, or neither.
46. [Clocks showing angles]
47. [Clocks showing angles]
48. [Clocks showing angles]

ARCHITECTURE  The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example of the indicated type of angle pair.
49. Supplementary angles  50. Vertical angles
51. Linear pair  52. Adjacent angles

53. ★ SHORT RESPONSE  Use the photograph shown at the right. Given that \(\angle PGB\) and \(\angle BGC\) are supplementary angles, and \(m \angle FGB = 120°\), explain how to find the measure of the complement of \(\angle BGC\).
Sample answer: Subtract 90° from \(m \angle FGB\).
54. **SHADOWS** The length of a shadow changes as the sun rises. In the diagram below, the length of \( CB \) is the length of a shadow. The end of the shadow is the vertex of \( \angle ABC \), which is formed by the ground and the sun’s rays. **Describe** how the shadow and angle change as the sun rises.

55a. \( y_1 = 90 - x \), \( 0 < x < 90 \); \( y_2 = 180 - x \), \( 0 < x < 180 \); the measure of a complement must be less than 90° and the measure of its supplement must be less than 180°.

55. **MULTIPLE REPRESENTATIONS** Let \( x \) be an angle measure. Let \( y_1 \) be the measure of a complement of the angle and let \( y_2 \) be the measure of a supplement of the angle.

   a. **Writing an Equation** Write equations for \( y_1 \) as a function of \( x \), and for \( y_2 \) as a function of \( x \). What is the domain of each function? **Explain**.

   b. **Drawing a Graph** Graph each function and **describe** its range.

   See margin for art; \( 0 < y_1 < 90 \), \( 0 < y_2 < 180 \).

56. **CHALLENGE** The sum of the measures of two complementary angles exceeds the difference of their measures by 86°. Find the measure of each angle. **Explain** how you found the angle measures.

   47°, 43°. **Sample answer:** Solve the system: \( x + y = 90 \), \( x + y = 86 = x - y \).

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**ASSESS AND RETEACH**

**Daily Homework Quiz**

Also available online

1. \( \angle 1 \) and \( \angle 2 \) are supplementary. If \( m \angle 1 = 97° \), find \( m \angle 2 \). 83°

2. \( \angle 3 \) and \( \angle 4 \) are complementary angles. If \( m \angle 3 = 74° \), find \( m \angle 4 \). 16°

3. Find \( m \angle EFH \). 96°

4. Find \( m \angle ABC \). 36°

5. Is it possible to draw a figure that contains exactly one pair of vertical angles? **Explain**. **No**; once you have drawn a pair of vertical angles, you have drawn two pairs of opposite rays. This automatically gives another pair of vertical angles.

**Online Quiz**

Available at my.hrw.com

**Diagnosis/Remediation**

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

**Challenge**

Additional challenge is available in the Chapter Resource Book.
1.6 Classify Polygons

**Key Vocabulary**
- polygon
- side
- vertex
- convex
- concave
- n-gon
- equilateral
- equiangular
- regular

**Identifying Polygons**
In geometry, a figure that lies in a plane is called a **plane figure**. A **polygon** is a closed plane figure with the following properties:
1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is **vertices**. A polygon can be named by listing the vertices in consecutive order. For example, ABCDE and CDEAB are both correct names for the polygon at the right.

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called **nonconvex** or **concave**.

**Example 1** Identify polygons
Tell whether the figure is a polygon and whether it is convex or concave.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
<th>c.</th>
<th>d.</th>
</tr>
</thead>
</table>

**Solution**
- a. Some segments intersect more than two segments, so it is not a polygon.
- b. The figure is a convex polygon.
- c. Part of the figure is not a segment, so it is not a polygon.
- d. The figure is a concave polygon.

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**Common Core Standards**

- **CC.9-12.G.MG.4**: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. (preparation for)
- **CC.9-12.G.MG.1**: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
CLASSIFYING POLYGONS  A polygon is named by the number of its sides.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Type of polygon</th>
<th>Number of sides</th>
<th>Type of polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td>10</td>
<td>Decagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td>12</td>
<td>Dodecagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
<td>n</td>
<td>n-gon</td>
</tr>
</tbody>
</table>

The term \( n \)-gon, where \( n \) is the number of a polygon’s sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent.

In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.

**Example 2**  
Classify polygons

Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

a.  
\[ \text{hexagon} \]  

b.  
\[ \text{trapezoid} \]  

c.  
\[ \text{cross} \]  

**Solution**

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

**Guided Practice** for Examples 1 and 2

1. Sketch an example of a convex heptagon and an example of a concave heptagon. See margin.

2. Classify the polygon shown at the right by the number of sides. Explain how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.  
   **Quadrilateral;** they all have the same length; they are all right angles.

**Differentiated Instruction**

**Below Level** Tell the students that a polygon is concave if it is dented inward at one or more of its vertices. To help them remember this term, you may want to suggest that they make a mental connection with the words **cavity** and **cave**.

See also the Differentiated Instruction Resources for more strategies.
Extra Example 3
A rack for billiard balls is shaped like an equilateral triangle. Find the length of a side. 14 in.

Key Question Example 3
- What is the distance around the table? 180 in.

Closing the Lesson
Have students summarize the major points of the lesson and answer the Essential Question: How do you classify polygons?

A polygon is a plane figure formed by three or more line segments. Each side intersects exactly two other sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

A polygon is convex if no line that contains a side of the polygon contains a point in the interior. Otherwise, the polygon is concave.

A polygon is regular if all sides are congruent and all angles in the interior are congruent.

The most basic way of classifying a polygon is by the number of sides. You can also tell whether the polygon is convex or concave, or indicate whether all the sides or angles are congruent. If all the sides of a convex polygon are congruent and all the angles are congruent, the polygon is a regular polygon.

2. Yes, the string will match the sides of a convex polygon, so it will be the perimeter of the polygon; no, the length of the string will be less than the perimeter of the polygon.

3. polygon; concave

4. Not a polygon; part of the figure is not a segment.

5. polygon; convex

6. Not a polygon; some segments intersect more than two segments.

Example 3
Find side lengths

Algebra A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.

Solution
First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

\[ 3x + 6 = 4x - 2 \quad \text{Write equation.} \]
\[ 6 = x + 2 \quad \text{Subtract } 3x \text{ from each side.} \]
\[ 8 = x \quad \text{Add } 2 \text{ to each side.} \]

Then find a side length. Evaluate one of the expressions when \( x = 8 \).

\[ 3x + 6 = 3(8) + 6 = 30 \]

The length of a side of the table is 30 inches.

Guided Practice for Example 3
3. The expressions \( 8y^\circ \) and \( (9y - 15)^\circ \) represent the measures of two of the angles in the table in Example 3. Find the measure of an angle. 120°

1.6 Exercises

Skill Practice

A

1. Vocabulary Explain what is meant by the term \( n \)-gon.

An \( n \)-gon is a polygon with \( n \) sides.

2. Writing Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? Explain. See margin.

Identifying Polygons Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is convex or concave. 3–8. See margin.

3. 4. 5. 6.

7. Multiple Choice Which of the figures is a concave polygon? C

A B C D
**Example 2** for Exs. 8–14

**Classifying** Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. **Explain** your reasoning. 8–13. See margin.

8.  
9.  
10.  

11.  
12.  
13.  

**Example 3** for Exs. 15–17

**Error Analysis** Two students were asked to draw a regular hexagon, as shown below. **Describe** the error made by each student.

**Student A:** the hexagon is concave, **Student B:** the hexagon does not have congruent sides.

**Student A**  
**Student B**

**Example 3** for Exs. 15–17

**Algebra** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon. 8 in.

16. **Algebra** The expressions $(9x + 5)^\circ$ and $(11x - 25)^\circ$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon. 140°

17. **Algebra** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side. 3 ft

**Using Properties** Tell whether the statement is **always**, **sometimes**, or **never** true.

18. A triangle is convex. **Always**

19. A decagon is regular. **Sometimes**

20. A regular polygon is equiangular. **Always**

21. A circle is a polygon. **Never**

22. A polygon is a plane figure. **Always**

23. A concave polygon is regular. **Never**

**Drawing** Draw a figure that fits the description. 24–27. See margin.

24. A triangle that is not regular

25. A concave quadrilateral

26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

**Algebra** Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of $x$.

28. \[ \begin{array}{c} x^2 + x \\ x^2 + 4 \end{array} \]

29. \[ \begin{array}{c} x^2 + 3x \\ x^2 + x + 2 \end{array} \]

30. \[ \begin{array}{c} x^2 + 2x + 40 \\ x^2 - x + 190 \end{array} \]

8. Octagon; regular; it has 8 congruent sides and angles.

9. Pentagon; regular; it has 5 congruent sides and angles.

10. Triangle; regular; it has 3 congruent sides and angles.

11. Triangle; none of these; the sides and/or the angles are not all congruent.

12. Quadrilateral; equilateral; it has 4 congruent sides.

13. Quadrilateral; equiangular; it has 4 congruent angles.

24–27. Sample answers are given.

24.

25.

27.
31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. **Explain** your reasoning. See margin.

32. **ARCHITECTURE** Longwood House, shown at the beginning of this lesson, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.

a. Tell whether the red polygon in the diagram is convex or concave. **convex**

b. Classify the red polygon and tell whether it appears to be regular.

**EXAMPLE 2** for Exs. 33–36

33. triangle; regular
34. quadrilateral; equiangular
35. octagon; regular
36. dodecagon; none of these

**SIGNs** Each sign suggests a polygon. Classify the polygon by the number of sides. **Tell whether it appears to be equilateral, equiangular, or regular.**

37. **MULTIPLE CHOICE** Two vertices of a regular quadrilateral are A(0, 4) and B(0, −4). Which of the following could be the other two vertices? **C**

- A. C(4, 4) and D(4, −4)
- B. C(−4, 4) and D(−4, −4)
- C. C(−8, 4) and D(8, 4)
- D. C(0, 8) and D(0, −8)

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850. **38a-b. See margin.**

a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.

b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.
39. **SHORT RESPONSE** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain.*

40. **EXTENDED RESPONSE** A segment that joins two nonconsecutive vertices of a polygon is called a diagonal. For example, a quadrilateral has two diagonals, as shown below.

<table>
<thead>
<tr>
<th>Type of polygon</th>
<th>Diagram</th>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Pentagon</td>
<td></td>
<td>?</td>
<td>? 5</td>
</tr>
<tr>
<td>Hexagon</td>
<td></td>
<td>?</td>
<td>? 6</td>
</tr>
<tr>
<td>Heptagon</td>
<td></td>
<td>?</td>
<td>? 7</td>
</tr>
</tbody>
</table>

a. Copy and complete the table. Describe any patterns you see.

b. How many diagonals does an octagon have? *Explain.

c. The expression \(\frac{n(n-3)}{2}\) can be used to find the number of diagonals in an n-gon. Find the number of diagonals in a 60-gon. 1710 diagonals

41. **LINE SYMMETRY** A figure has line symmetry if it can be folded over exactly onto itself. The fold line is called the line of symmetry. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

a. A regular triangle  b. A regular pentagon  c. A regular hexagon  d. A regular octagon

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge. *See margin.*

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**QUIZ**

In each diagram, \(\overline{BD}\) bisects \(\angle ABC\). Find \(m\angle ABD\) and \(m\angle DBC\).

1. \((x + 20)^\circ\)  
2. \((10x - 42)^\circ\)  
3. \((18x + 27)^\circ\)

Find the measure of (a) the complement and (b) the supplement of \(\angle 1\).

4. \(m\angle 1 = 47^\circ\)  
5. \(m\angle 1 = 19^\circ\)  
6. \(m\angle 1 = 75^\circ\)  
7. \(m\angle 1 = 2^\circ\)

Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is convex or concave.

8. \(\overline{\text{polygon; concave}}\)  
9. \(\overline{\text{polygon; convex}}\)  
10. \(\overline{\text{polygon}}\)

See **EXTRA PRACTICE** in Student Resources  **ONLINE QUIZ** at my.hrw.com

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Online Quiz

Available at my.hrw.com

**Diagnosis/Remediation**

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

**Challenge**

Additional challenge is available in the Chapter Resource Book.

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40a. See margin for art. *Sample answer: The difference between successive number of diagonals is increasing by one.*

42. See Additional Answers.
Alternative Strategy
Example 2 in the previous lesson was solved by relying on the markings on the geometric figures. In this example, students measure the figures using informal tools.

Teaching Strategy
Students are generally discouraged from relying on the appearance of congruence or similarity of figures, but this method demonstrates that it is sometimes appropriate to take your own measurements to solve problems.

Another Way to Solve Example 2

**MULTIPLE REPRESENTATIONS** In Example 4, you saw how to use markings on geometric figures to determine whether sides and angles are congruent. Using measuring tools can allow you to decide whether measures in a diagram are equal.

**Problem**
Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

**Method 1**
Using a Measuring Tool For problems in your Geometry book, you are expected to rely on the markings on the diagrams to determine whether sides and angles are congruent, but you can also use a piece of paper to test whether lengths and angles in a diagram are equal.

**Step 1** Choose a Tool to Measure Length Because it can be hard to use a ruler on a diagram in your book, you can copy the diagram onto a piece of paper. Or you can use a piece of paper to measure lengths by marking a side length on a scrap of paper and then comparing to the other sides.

**Step 2** Choose a Tool to Measure Angle Copy the diagram onto a piece of paper so you can use a protractor. Or you can use a piece of paper to measure angles by folding a scrap of paper to match an angle measure and then comparing to the other angles.

**Practice**
Choose a tool to use to measure the lengths and angles of each diagram. Tell whether the figure is equilateral, equiangular, or regular.

1. regular

2. none of these

3. regular

4. equiangular
1. **MULTI-STEP PROBLEM** You are covering the rectangular roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost $0.75 per square foot and wood shingles cost $1.15 per square foot.

   a. Find the area of the roof in square feet. 108 ft²
   
   b. Find the cost of using asphalt shingles and the cost of using wood shingles. $81, $124.20
   
   c. About how much more will you pay to use wood shingles for the roof? $43.20

2. **OPEN-ENDED** In the window below, name a convex polygon and a concave polygon. Classify each of your polygons by the number of sides. Sample answer: **GBCDFEA; triangle, heptagon**

3. **EXTENDED RESPONSE** The diagram shows a decoration on a house. In the diagram, \( \angle HGD \) and \( \angle HGF \) are right angles, \( m \angle DGB = 21^\circ \), \( m \angle HGB = 55^\circ \), \( \angle DGB \) is supplementary to \( \angle FGC \), and \( \angle HGB \) is supplementary to \( \angle HCG \).

   a. List two pairs of complementary angles and five pairs of supplementary angles.
   
   b. Find \( m \angle FGC \), \( m \angle BGH \), and \( m \angle HGC \). **Explain** your reasoning.
   
   c. Find \( m \angle HCG \), \( m \angle DBG \), and \( m \angle FCG \). **Explain** your reasoning.

4. **GRIDDED ANSWER** \( \angle 1 \) and \( \angle 2 \) are supplementary angles, and \( \angle 1 \) and \( \angle 3 \) are complementary angles. Given \( m \angle 1 \) is 28° less than \( m \angle 2 \), find \( m \angle 3 \) in degrees. 14°

5. **EXTENDED RESPONSE** You use bricks to outline the borders of the two gardens shown below. Each brick is 10 inches long.

   a. You lay the bricks end-to-end around the border of each garden. How many bricks do you need for each garden? **Explain**.
   
   b. The bricks are sold in bundles of 100. How many bundles should you buy? **Explain**.
   
   3 bundles; 206 bricks are needed, thus 3 bundles.

6. **SHORT RESPONSE** The frame of a mirror is a regular pentagon made from pieces of bamboo. Use the diagram to find how many feet of bamboo are used in the frame.

   7.5 ft of bamboo

7. **GRIDDED ANSWER** As shown in the diagram, a skateboarder tilts one end of a skateboard. Find \( m \angle ZWX \) in degrees. 37°

8. **SHORT RESPONSE** Use the diagram below.

   a. Find the perimeter of quadrilateral \( ABCD \).
   
   b. Find the area of triangle \( ABC \) and the area of triangle \( ADC \). What is the area of quadrilateral \( ABCD \)? **Explain**.

   8 square units, 8 square units; 16 square units; the area of the quadrilateral is the sum of the areas of the two triangles.
**BIG IDEAS**

**Describing Geometric Figures**
You learned to identify and classify geometric figures.

<table>
<thead>
<tr>
<th>Point $A$</th>
<th>Line $AB$ (or $\overline{AB}$)</th>
<th>Plane $M$</th>
<th>Segment $AB$ (or $\overline{AB}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$A$ to $B$</td>
<td>$M$</td>
<td>$A$ to $B$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ray $AB$ (or $\overline{AB}$)</th>
<th>Angle $A$ (or $\angle A$, $\angle BAC$, or $\angle CAB$)</th>
<th>Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ to $B$</td>
<td>$A$ to $B$ of $\angle A$, $\angle BAC$, or $\angle CAB$</td>
<td>Quadrilateral $ABCD$, Pentagon $PQRST$</td>
</tr>
</tbody>
</table>

**Measuring Geometric Figures**

**SEGMENTS** You measured segments in the coordinate plane.

- **Distance Formula**
  Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:
  \[ AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

- **Midpoint Formula**
  Coordinates of midpoint $M$ of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:
  \[ M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

**ANGLES** You classified angles and found their measures.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- Complementary angles $\measuredangle 1 + \measuredangle 2 = 90^\circ$
- Supplementary angles $\measuredangle 3 + \measuredangle 4 = 180^\circ$

**Understanding Equality and Congruence**
Congruent segments have equal lengths. Congruent angles have equal measures.

- $\overline{AB} = \overline{BC}$ and $\overline{AB} = \overline{BC}$
- $\angle JKL = \angle LKM$ and $m\angle JKL = m\angle LKM$
Extra Example 1
Use the diagram shown.

a. Give another name for \( \overrightarrow{XY} \). \( \overrightarrow{XW}, \overrightarrow{YW}, \overrightarrow{WX}, \overrightarrow{WY}, \text{ or } \overrightarrow{YX} \)
b. Name three points that are collinear. \( X, W, Y \)
c. Name four points that are coplanar. \( X, W, Y, Q \)

2. Sample:

\[ \text{A} \quad \text{B} \quad \text{C} \]

REVIEW KEY VOCABULARY

- undefined terms
- point, line, plane
- collinear, coplanar points
- defined terms
- line segment, endpoints
- ray, opposite rays
- intersection
- postulate, axiom
- coordinate
- distance
- between
- congruent segments
- midpoint
- segment bisector
- angle
- sides, vertex, measure
- acute, right, obtuse, straight
- congruent angles
- angle bisector
- construction
- complementary angles
- supplementary angles
- adjacent angles
- linear pair
- vertical angles
- polygon
- side, vertex
- convex, concave
- \( n \)-gon
- equilateral, equiangular, regular

VOCABULARY EXERCISES

1. Copy and complete: Points \( A \) and \( B \) are the _____ of \( \overline{AB} \). endpoints

2. Draw an example of a linear pair. See margin.

3. If \( Q \) is between points \( P \) and \( R \) on \( \overrightarrow{PR} \), and \( PQ = QR \), then \( Q \) is the _____ of \( PR \). midpoint

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this chapter.

1.1 Identify Points, Lines, and Planes

Example

Use the diagram shown at the right.

Another name for \( \overrightarrow{CD} \) is line \( m \).

Points \( A, B, \) and \( C \) are collinear.

Points \( A, B, C, \) and \( F \) are coplanar.

Exercises

4. Give another name for line \( g \). Sample answer: \( \overrightarrow{YZ} \)

5. Name three points that are not collinear. Sample answer: \( P, Y, Z \)

6. Name four points that are coplanar. Sample answer: \( N, X, V, Z \)

7. Name a pair of opposite rays. \( \overrightarrow{YZ}, \overrightarrow{XY} \)

8. Name the intersection of line \( h \) and plane \( M \). point \( Y \)
1.2 Use Segments and Congruence

**Example**

Find the length of $HJ$.

$GJ = GH + HJ$  \hspace{2cm} \text{Segment Addition Postulate} \\
27 = 18 + HJ \hspace{2cm} \text{Substitute 27 for } GJ \text{ and } 18 \text{ for } GH. \\
9 = HJ \hspace{2cm} \text{Subtract 18 from each side.}

**Exercises**

Find the indicated length.

9. Find $AB$. 12
10. Find $NP$. 30
11. Find $XY$. 7

![Diagram](image)

12. The endpoints of $DE$ are $D(-4, 11)$ and $E(-4, -13)$. The endpoints of $GH$ are $G(-14, 5)$ and $H(-9, 5)$. Are $DE$ and $GH$ congruent? Explain. No; $DE = 24$, $GH = 5$

1.3 Use Midpoint and Distance Formulas

**Example**

$\overline{EF}$ has endpoints $E(1, 4)$ and $F(3, 2)$. Find (a) the length of $\overline{EF}$ rounded to the nearest tenth of a unit, and (b) the coordinates of the midpoint $M$ of $\overline{EF}$.

a. Use the Distance Formula.

$EF = \sqrt{(3 - 1)^2 + (2 - 4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} = 2.8 \text{ units}$

b. Use the Midpoint Formula.

$M \left(\frac{1 + 3}{2}, \frac{4 + 2}{2}\right) = M(2, 3)$

**Exercises**

2, 3, and 4

13. Point $M$ is the midpoint of $\overline{JK}$. Find $JK$ when $JM = 6x - 7$ and $MK = 2x + 3$. 16

In Exercises 14–17, the endpoints of a segment are given. Find the length of the segment rounded to the nearest tenth. Then find the coordinates of the midpoint of the segment.

14. $A(2, 5)$ and $B(4, 3)$ \hspace{2cm} 2.8; (3, 4)
15. $F(1, 7)$ and $G(6, 0)$ \hspace{2cm} 8.6; (3.5, 3.5)
16. $H(-3, 9)$ and $J(5, 4)$ \hspace{2cm} 9.4; (1.65)
17. $K(10, 6)$ and $L(0, -7)$ \hspace{2cm} 16.4; (5, -0.5)

18. Point $C(3, 8)$ is the midpoint of $\overline{AB}$. One endpoint is $A(-1, 5)$. Find the coordinates of endpoint $B$. (7, 11)

19. The endpoints of $\overline{EF}$ are $E(2, 3)$ and $F(8, 11)$. The midpoint of $\overline{EF}$ is $M$. Find the length of $\overline{EM}$. 5
1.4 Measure and Classify Angles

Example
Given that $m \angle YXV$ is $60^\circ$, find $m \angle YXZ$ and $m \angle ZXV$.

**STEP 1** Find the value of $x$.

$$m \angle YXV = m \angle YXZ + m \angle ZXV$$

$$60^\circ = (2x + 11)^\circ + (x + 13)^\circ$$

$$x = 12$$

**STEP 2** Evaluate the given expressions when $x = 12$.

$$m \angle YXZ = (2x + 11)^\circ = (2 \cdot 12 + 11)^\circ = 35^\circ$$

$$m \angleZXV = (x + 13)^\circ = (12 + 13)^\circ = 25^\circ$$

Exercises

20. In the diagram shown at the right, $m \angle LMN = 140^\circ$. Find $m \angle PMN$. 50°

21. $\overline{VZ}$ bisects $\angle UVW$, and $m \angle UVZ = 81^\circ$. Find $m \angle UVW$. Then classify $\angle UVW$ by its angle measure. 162°, obtuse

1.5 Describe Angle Pair Relationships

Example
a. $\angle 1$ and $\angle 2$ are complementary angles. Given that $m \angle 1 = 37^\circ$, find $m \angle 2$.

$$m \angle 2 = 90^\circ - m \angle 1 = 90^\circ - 37^\circ = 53^\circ$$

b. $\angle 3$ and $\angle 4$ are supplementary angles. Given that $m \angle 3 = 106^\circ$, find $m \angle 4$.

$$m \angle 4 = 180^\circ - m \angle 3 = 180^\circ - 106^\circ = 74^\circ$$

Exercises

22. $m \angle 1 = 12^\circ$ 78°

23. $m \angle 1 = 83^\circ$ 7°

24. $m \angle 1 = 46^\circ$ 44°

25. $m \angle 1 = 2^\circ$ 88°

26. $m \angle 3 = 116^\circ$ 64°

27. $m \angle 3 = 56^\circ$ 124°

28. $m \angle 3 = 89^\circ$ 91°

29. $m \angle 3 = 12^\circ$ 168°

30. $\angle 1$ and $\angle 2$ are complementary angles. Find the measures of the angles when $m \angle 1 = (x - 10)^\circ$ and $m \angle 2 = (2x + 40)^\circ$. 10°, 80°

31. $\angle 1$ and $\angle 2$ are supplementary angles. Find the measures of the angles when $m \angle 1 = (3x + 50)^\circ$ and $m \angle 2 = (4x + 32)^\circ$. Then classify $\angle 1$ by its angle measure. 92°, 88°, obtuse
Classify Polygons

**Example**

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. Explain.

The polygon has four sides, so it is a quadrilateral. It is not equiangular or equilateral, so it is not regular.

**Exercises**

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. Explain. 32–34. See margin.

32.  

33.  

34.  

35. Pentagon $ABCDE$ is a regular polygon. The length of $BC$ is represented by the expression $5x - 4$. The length of $DE$ is represented by the expression $2x + 11$. Find the length of $AB$. 21

Extra Example 6

Classify the polygon shown by the number of sides and decide whether it is a regular polygon.

hexagon, regular

32. Triangle; regular; it has three congruent sides and three congruent angles.

33. Quadrilateral; equiangular; it has four congruent angles but its four sides are not all congruent.

34. Octagon; equilateral; it has eight congruent sides but its eight angles are not all congruent.