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Marketing Science Institute
1000 Massachusetts Avenue
Cambridge, MA USA
02138-5396

Phone: 617.491.2060
Fax: 617.491.2065
www.msi.org

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A New Approach to Modeling the Adoption of New Products: Aggregated Diffusion Models

Olivier Toubia, Jacob Goldenberg, and Rosanna Garcia

This study combines a traditional aggregate approach with an agent-based approach to develop a model that more accurately describes and forecasts the diffusion of new products.

Olivier Toubia is the David W. Zalaznick Associate Professor of Business, Graduate School of Business, Columbia University. **Jacob Goldenberg** is Senior Lecturer of Marketing, School of Business Administration, Hebrew University. **Rosanna Garcia** is Assistant Professor of Marketing, College of Business Administration, Northeastern University.

Report Summary

Companies need tools and models that accurately describe and forecast the diffusion process of new products. Most tools for predicting the diffusion of an innovation, given early data, use aggregate models (such as the Bass model) that capture the impact of advertising and word-of-mouth on adoption. Because of their aggregate nature, however, (i.e., these models treat each month or each year as one observation), they are often unable to produce reliable predictions early in the diffusion process, when such predictions would be most helpful (e.g., in order to adjust the launch campaign). Moreover, model assumptions about the influence of social interactions on individual consumers are often not well defined, and it is often unclear when a given model is appropriate.

This study addresses these limitations by combining the traditional aggregate approach with the agent-based approach. Toubia, Goldenberg, and Garcia's proposed models are based on explicit assumptions about the adoption process, and model parameters are expressed as functions of well-defined, measurable variables describing individual consumer behavior and network interactions.

They demonstrate how their approach enabled a major consumer packaged goods (CPG) manufacturer to develop a theory-driven aggregate diffusion model and to calibrate this model shortly after launch using survey data.

The proposed approach addresses the limitations of aggregate diffusion models on three fronts. First, it allows developing rich models that are based on explicit assumptions about the adoption process. Second, because the parameters of the proposed models are linked to well-defined, measurable variables describing individual consumer behavior and network interactions, these models may be calibrated using survey data shortly after launch. Third, because they are based on a set of explicit assumptions, the authors are able to provide guidelines to help managers and researchers identify conditions under which they provide an accurate description of the diffusion process, and conditions under which they should be replaced with alternative diffusion models. ■

Introduction

The launch of new products or services is a sensitive and critical activity which embodies some of the highest financial risks for firms (Robertson 1971). Investment commitments in new product development increase as the process advances, peaking at launch. Therefore, companies have a strong need for tools and models that accurately describe and forecast the diffusion process of new products. Two methodologies have emerged from diffusion research in marketing: aggregate (market-level) approaches and disaggregate (individual-level) approaches.

Aggregate models forecast the diffusion curve without direct evaluation of an individual's adoption decision. The Bass model (Bass 1969) and its variants are the most prominent models in this stream of research. The basic Bass model predicts category-level adoption by capturing two drivers of adoption: "external" marketing forces (such as advertising), and "internal" social interaction (or word-of-mouth) effects. While parsimonious, the Bass model suffers from several shortcomings. For example, it does not capture a set of clearly identified assumptions on how and why consumers adopt new products (Chatterjee and Eliashberg 1990; Mahajan, Muller, and Bass 1990; Mahajan and Wind 1986). In particular, its parameters do not have a meaningful, measurable definition (e.g., the probability of an event happening or the amount of a given quantity). Moreover, while the Bass model usually fits observed diffusion curves fairly well, it is not as useful for *predicting* the diffusion of an innovation before or shortly after its launch (see, for example, Hauser, Tellis, and Griffin 2006; Kohli, Lehmann, and Pae 1999; Mahajan, Muller, and Bass 1990; Srinivasan and Mason 1986; Van den Bulte and Lilien 1997). Finally, it is not always obvious when (e.g., for which types of innovation, which type of consumer network) the Bass model or one of its variants is appropriate (Mahajan and Muller 1979; Mahajan, Muller, and Bass 1990; Mahajan and Wind 1986).

Disaggregate models, on the other hand, study the diffusion of an innovation using the individual customer as the unit of analysis. One important type of disaggregate models are agent-based models (Garber et al. 2004; Goldenberg, Libai, and Muller 2002), which capture similar "internal" and "external" forces as the Bass model. One of their advantages is that they allow a wide range of alternative assumptions on the diffusion process, such as various network effects or the existence of influential customers.

However, one of their limitations is that the aggregate diffusion process is not easily quantifiable through simple formulas like the Bass model equation. Instead, inference has relied on computer simulations that require using specialized software in which several parameters (e.g., size and structure of the customer network) have to be set by the researcher. The selection of these parameters may be somewhat arbitrary and multiple factors may influence the results. Because of all these limitations, to the best of our knowledge agent-based models have never been used to forecast the entire diffusion process of an innovation.

In this paper we attempt to combine the benefits of aggregate and disaggregate diffusion models while avoiding most of their respective shortcomings. We propose a class of aggregate diffusion models built as an approximation of a general agent-based model. The proposed models are based on explicit assumptions regarding the adoption process, and model parameters are expressed as functions of well-defined, measurable variables describing individual consumer behavior and network interactions. The Bass model is one special case of this class of models.

The proposed approach allows us to address the limitations of aggregate diffusion models on three fronts. First, it allows us to develop rich models that are based on explicit assumptions regarding the adoption process. Second, because the parameters of the proposed models are linked to well-defined, measurable variables describing individual consumer behavior and network interactions, these models may be calibrated using survey data shortly after launch. Third, because they are based on a set of explicit assumptions, we are able to provide guidelines to help managers and researchers identify some conditions under which they provide an accurate description of the diffusion process, and some conditions under which they should be replaced with alternative diffusion models.

This paper is organized as follows. We briefly review agent-based models in the next section, then introduce our class of aggregate diffusion models. We then provide guidelines to calibrate these models using survey data collected shortly after launch. We also provide guidelines to identify some conditions under which these models are good descriptors of the diffusion process and some conditions under which they should be replaced with alternative models, and illustrate these guidelines by identifying such conditions for the Bass model. We then show how we helped a major consumer packaged goods manufacturer develop an aggregate diffusion model and calibrate this model by surveying a panel of consumers shortly after launch. Finally, we conclude and offer directions for future research.

Brief Review of Agent-based Models

The way in which information spreads in a given social system may be described as an *adaptive complex system*, or a system that consists of a large number of individual entities that interact with each other, ultimately generating large-scale, collective, visible behavior (Waldorp 1992). A broad set of tools labeled *complex system methods* has emerged in recent decades to study such systems. This tool set, which originated in physics and biology, has moved in recent years into the social sciences (Hegselmann 1998), including economics (Rosser 1999; Tesfatsion 2006), management (Anderson 1999), and marketing (Garber et al. 2004; Goldenberg, Libai, and Muller 2002).

We review here the agent-based model used by Garber et al. (2004) and Goldenberg, Libai, and Muller (2002), on which our model is based. Each agent is a potential adopter of the innovation. The probability that a given agent adopts the innovation at time $t + 1$ (given that it has not adopted yet) depends on two factors: external marketing forces (such as advertising), represented by a parameter p, and internal social interaction (or word-of-mouth) effects, represented by a parameter q. These parameters capture the same forces as the parameters of the Bass model, although we will see later that the parameter q of an agent-based model is not directly comparable to the parameter q in the Bass model. Garber et al. (2004) and Goldenberg, Libai, and Muller (2002) use the following specification for the probability that agent i adopts in period $(t + 1)$:

$$P(adopt_{i,(t+1)} \mid \text{not adopted yet}) = 1 - (1 - p)(1 - q)^{n_{it}} \quad (1)$$

where n_{it} is the number of agents connected to agent i that have already adopted at time t. In that case q may be interpreted as the probability of adopting based on one encounter with an adopter. The probability of adopting is equal to one minus the probability of "resisting" the innovation in period $(t + 1)$, which is equal to the probability of "resisting" the external forces and "resisting" n_{it} influences by previous adopters.

One of the advantages of the agent-based approach is that Equation 1 may be modified to capture a wide range of alternative assumptions on the diffusion process, such as various network effects or the existence of influential consumers. However, one of the current limitations of the agent-based approach is that the aggregate diffusion process is not available in closed form. Instead, inference has relied on simulations of populations of agents. Such

Table 1
List of variables

Type of variable	Variable name	Definition
Input	L	Number of different types of relevant social interactions
	m	Market potential
	$ties_i^l$	Number of ties of type l for agent i
	p_i	Coefficient capturing the effect of external forces for agent i
	q_i^l	Probability of adopting based on one relevant social interaction of type l for agent i
	a_τ^l	Probability that an agent j who adopted the innovation in period $t - \tau$ engages in a relevant social interaction of type l in period t with any of its ties
Output	n_{it}^l	Number of agents with whom agent i has a relevant social interaction of type l in period t (by definition, $n_{it}^l \leq ties_i^l$)
	S_t	Number of adopters at the end of period t

simulations require the use of specialized software and require setting several parameters exogenously (e.g., size and structure of the network), allowing multiple factors to influence the results.

Constructing Aggregate Diffusion Models from Disaggregate Models

In this section we show how a class of aggregate diffusion models may be constructed from a disaggregate agent-based model, which is a generalized version of the model used by Garber et al. (2004) and Goldenberg, Libai, and Muller (2002). We summarize the parameters of the model in Table 1. We assume that there are L different types of social interactions that are relevant to the innovation under study. These different types may reflect interactions between different types of agents, or different types of interactions between agents of the same type (e.g., strong ties versus weak ties). Different types of agents may be used, for example, to reflect various relationships (e.g., colleagues, relatives, or friends), or different predispositions towards the innovation (e.g., influentials versus imitators, as in Van den Bulte and Joshi 2007). For example, in the model used in our managerial application, we used $L = 2$ to capture the existence of two different types of recommendations in the oral care category (recommendations from dental professionals versus recommendations from other consumers). We assume that agent i is connected to a number $ties_i^l$ of agents of type l. A connection does not necessarily lead to a relevant social interaction at each period (e.g., a consumer may meet a friend but not discuss the innovation), but a connection has to exist for a relevant social interaction to take place between two agents.

We assume that the probability of agent i adopting at time $(t + 1)$ given that the agent has not adopted yet is given by:

$$P(adopt_{i,(t+1)} \mid \text{not adopted yet}) = [1 - (1 - p_i)(1 - q_i^1)^{n_{it}^1} \ldots (1 - q_i^L)^{n_{it}^L}] \quad (2)$$

where:

- n_{it}^l is the number of agents with whom agent i has a relevant social interaction of type l in period t.
- $q_i^1 \ldots q_i^L$ reflect the probabilities of adopting based on each type of relevant social interaction.

Equation 2 may be interpreted similarly to Equation 1, i.e., the probability of adopting is equal to one minus the probability of "resisting" the innovation in period $(t + 1)$,[1] which is equal to the probability of "resisting" the external forces and "resisting" n_{it}^l social interactions of each type.

We assume that if agent j, who has adopted the product in period $t - \tau$, is connected to agent i in period t, this connection will result in a relevant social interaction of type l with probability a_τ^l. For example, if we assume that the relevant social interactions are recommendations, a_τ^l represents the probability that agent i recommends the product to agent j. If we assume that social influences work through observing other consumers using the product, a^l_τ represents the probability that agent *i* will be using the product when it encounters agent *j*. Note that Garber et al. (2004) and Goldenberg, Libai, and Muller (2002) implicitly assume that a^l_τ is always equal to 1. Note also that the model may be easily extended to make all the parameters a function of time. We leave such extension to future research.

Finally, we make the assumption that the ties between agents are randomly formed again at each period. This last technical assumption is used to obtain closed-form expressions of the first-order approximation of the diffusion process. In particular, it allows writing the number of future adopters as a function of the *number* of previous adopters, ignoring their identities. We will explore violations of this assumption in the next section, and show, for example, that the Bass model is robust to violations of this assumption, as long as the number of ties per agent is large enough.

We now study the diffusion process implied by the above assumptions. Let S_t be the number of adopters at the end of period *t*, and *m* be the total market size (i.e., total number of agents). Under the above set of assumptions, n^l_{it} is a random variable with a binomial distribution:

$$B\left(ties^l_i, a_0 \frac{S_t - S_{t-1}}{m} + a_1 \frac{S_{t-1} - S_{t-2}}{m} + \ldots + a_{t-1} \frac{S_1}{m}\right).$$

Indeed, the probability that a randomly drawn agent *j* (from the set that excludes agent *i*) has already adopted the innovation in period *t* and engages in a relevant social interaction with agent *i* in period *t* is:

$$\begin{aligned}&\text{Prob}(j \text{ has already adopted}, j \text{ engages in} \\ &\quad \text{relevant social interaction at } t) = \\ &\quad \sum_{\tau=0}^{t-1} \text{Prob}(j \text{ adopted at } t-\tau, j \text{ engages in} \\ &\quad \text{relevant social interaction at } t) = \\ &\quad \sum_{\tau=0}^{t} \frac{S_{t-\tau} - S_{t-\tau-1}}{m-1} \cdot a^l_\tau.\end{aligned}$$

We show the following proposition (we refer the reader to Technical Appendix A at http://www.msi.org/techapp/08-103 for a proof):

Proposition: As the number of agents goes to infinity, the first order approximation in p_i and $\{q^l_i\}$ of the aggregate diffusion pattern implied by Equation 2 converges to:

$$\frac{S_{t+1}}{m} \approx \frac{S_t}{m} + \left(1 - \frac{S_t}{m}\right)\left[\frac{\sum_{i=1}^{m} p_i}{m} + \sum_{l=1}^{L} \frac{\sum_{i=1}^{m} ties^l_i \cdot q^l_i}{m} \cdot \left(a^l_0 \frac{S_t - S_{t-1}}{m} + a^l_1 \frac{S_{t-1} - S_{t-2}}{m} + \ldots + a^l_{t-1} \frac{S_1}{m}\right)\right] \quad (3)$$

Equation 3 defines a class of aggregate models. Alternative assumptions on the adoption process, reflected by p_i, L, $ties^l_i$, q^l_i, and a^l_τ, give rise to alternative special cases of this class of models. We believe that constructing (or reconstructing) aggregate models in this manner provides several benefits, which we illustrate in the remainder of this paper. First, it enables capturing a wide range of assumptions on the adoption process. Second, because the parameters of these aggregate models are explicit functions of a set of well-defined variables that describe the behavior of individual consumers and their interactions, these models may be calibrated shortly after launch using survey data. Third, conditions under which these models are accurate may be obtained by studying violations of the (explicit) assumptions on which these models are built.

The Bass model as a special case

For the remainder of the paper, we drop an index on a parameter when the parameter is constant for all values of that index. Let us first consider the special case in which $L = 1$, and a_τ is constant for all τ and equal to a. In such a case, only one type of social interaction is assumed to influence the diffusion process, and the probability of an adopter engaging in a relevant social interaction with one of its ties is constant across periods. Equation 3 then reduces to:

$$\frac{S_{t+1}}{m} \approx \frac{S_t}{m} + \left(1 - \frac{S_t}{m}\right)\left[\frac{\sum_{i=1}^{m} p_i}{m} + \frac{\sum_{i=1}^{m} ties_i \cdot q_i}{m} \cdot a \cdot \frac{S_t}{m}\right]$$

Therefore, the Bass model

$$\frac{S_{t+1}}{m} = \frac{S_t}{m} + \left(p^{Bass} + q^{Bass}\frac{S_t}{m}\right) \cdot \left(1 - \frac{S_t}{m}\right)$$

is a special case of Equation 3, with

$$p^{Bass} = \frac{\sum_{i=1}^{m} p_i}{m}$$

and

$$q^{Bass} = \frac{\sum_{i=1}^{m} ties_i \cdot a \cdot q_i}{m},$$

i.e., the innovation coefficient is equal to the average innovation coefficient across consumers and the diffusion coefficient is equal to the average of the product of the number of ties per consumer, the probability that a tie will lead to a relevant social interaction, and the probability of adopting based on one relevant social interaction. Note that this reconstruction of the Bass model expresses q^{Bass} as a function of a set of well-defined and measurable variables describing the adoption process.

Alternative theory-driven models

A wide range of aggregate diffusion models may be developed by considering alternative assumptions regarding the adoption process, reflected by the variables L, $ties_i^l$, p_i, q_i^l, and a_τ^l. For example, below in "A Managerial Application," we will consider a model with $L = 2$, which captures the difference between recommendations coming from consumers and recommendations coming from professionals.

Here we illustrate the construction of models that relax the assumption that a_τ^l, the probability of a relevant social interaction of type l occurring, is constant across τ. As noted above, assuming that a_τ^l is constant across τ is equivalent to assuming that agent i is equally likely to be influenced by any adopter to which it is connected, irrespective of when it has adopted. Instead, let us consider a product category in which social interactions work primarily through recommendations, and in which a consumer who adopts the innovation in period τ will recommend it on the same period and over the following $K - 1$ periods, such that $a_\tau = 0$ for $\tau \geq K$. In that case, Equation 3 becomes:

$$\frac{S_{t+1}}{m} = \frac{S_t}{m} + \left[\frac{\sum_{i=1}^{m} p_i}{m} + \frac{\sum_{i=1}^{m} ties_i \cdot q_i}{m} \cdot \left(a_0\frac{S_t - S_{t-1}}{m} + a_1\frac{S_{t-1} - S_{t-2}}{m} + \ldots + a_{K-1}\frac{S_{t-K+1} - S_{t-K}}{m}\right)\right] \cdot \left(1 - \frac{S_t}{m}\right).$$

Figure 1 illustrates the diffusion process implied by this aggregate model with

$$\frac{\sum_{i=1}^{m} p_i}{m} = .005,$$

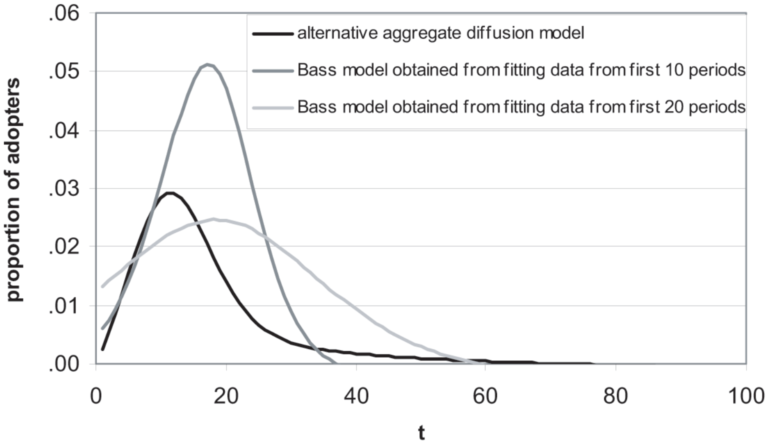
$q_i = .4$ for all i, $ties_i = 10$ for all i, $K = 5$, and exponentially decreasing recommendation probabilities: $a_0 = 1/5$, $a_1 = 1/10$, $a_2 = 1/20$, $a_3 = 1/40$, $a_4 = 1/80$. This model gives rise to a diffusion pattern that is different from the one implied by a constant parameter a_τ (i.e., the Bass model). In particular, Figure 1 also plots the Bass model obtained by fitting respectively the first 10 and 20 periods of the diffusion process (the peak occurs in period 14). This example illustrates that a change in the (explicit or implicit) assumptions on social interactions may have a great impact on the aggregate diffusion process.

Calibration Using Survey Data Collected Shortly after Launch

Calibrating the proposed models necessitates the estimation of the variables $\{p_i\}$, $\{q_i^l\}$, $\{ties_i^l\}$, and $\{a_\tau^l\}$. The best approach to the estimation of these variables depends on the context and on the specific assumptions reflected by these parameters (e.g., d_τ may capture recommendations in some cases and encounters with adopters in others). Therefore we only highlight a general approach in this section, which we apply below in "A Managerial Application" to a specific example.

Figure 1

Example of an Alternative Aggregate Diffusion Model



Estimating the parameters $ties^l_i$ and a^l_τ

Several approaches may be used to estimate the parameters $\{ties^l_i\}$ and $\{a^l_\tau\}$. For example, these parameters may be measured directly using panel data that track interactions between consumers. Alternatively, these parameters may be measured using survey data. In general, $ties^l_i$ may be measured by asking consumers about their number of interactions per period, and a^l_τ may be measured by asking those consumers who have already adopted the innovation about the number of relevant social interactions they have initiated in the periods following the adoption.

Estimating the parameters p_i and q^l_i

The parameters $\{p_i\}$ and $\{q^l_i\}$ may be estimated based on survey data that contain two pieces of information: (1) whether each respondent has adopted the innovation on or before the period of the survey, (2) the number of relevant social interactions of each type to which each respondent has been the target since the introduction of the innovation. Indeed, the probability that a consumer adopts the innovation at any time between t = 1 and t = T may be written as:

$$L_{iT}(\{p_i\},\{q^l_i\}) = 1 - \prod_{l=1}^{L} (1-p_i)^T \cdot (1-q^l_i)^{\sum_{t=1}^{T} n^l_{it}}.$$

The parameters $\{p_i\}$ and $\{q^l_i\}$ may then be estimated based on this likelihood function. In our managerial application we make the simplifying assumption that p_i and q_i are constant across consumers, and estimate p and $\{q^l\}$ by maximizing the likelihood function:

$$L(p,\{q^l\}) = \prod_{i=1}^{I} L_{iT}(p,\{q^l\}),$$

where I is the number of consumers in the survey. Future research may relax this assumption, and capture heterogeneity in the parameters using approaches such as hierarchical Bayes (Rossi and Allenby 2003) or latent classes (Jain, Vilcassim, and Chintagunta 1994).

Under What Conditions Are These Models Accurate?

When developing any model of diffusion, it is important to identify conditions under which this model is valid and useful, and conditions under which it should be replaced with an alternative model (Mahajan and Wind 1986). In this section we provide some guidelines to identify such conditions for any aggregate model developed using the approach proposed in this paper. Because the specific conditions will depend on the assumptions made by the specific model, we focus on proposing some general guidelines, which we illustrate on the Bass model. In Technical Appendix B at http://www.msi.org/techapp/08-103, we report the result of a similar analysis, applied to the model used below in "A Managerial Application."

The aggregate models proposed in this paper are first-order approximations of an agent-based model, under a specific set of assumptions which may be classified into two types. The first type is behavioral assumptions on the adoption process (reflected by the parameters L, $ties^l_i$, p_i, q^l_i, and a^l_τ), and the second type is the technical assumption that the ties between agents are randomly reformed at each period.

The technical assumption is made in order to derive closed-form expressions of the aggregate diffusion pattern. Therefore, there exist three major sets of potential conditions under which the aggregate models proposed in this paper may be inaccurate: (1) the behavioral assumptions are incorrect; (2) the technical assumption is incorrect; (3) the behavioral and technical assumptions are correct, but the first-order approximation is a poor approximation of the diffusion process.

We have already illustrated in the previous section that incorrect behavioral assumptions may lead to inaccurate models. We assume here that the researcher who developed the model believes that the behavioral assumptions are correct, and propose that such a researcher should systematically study the impact of violating the technical assumption on which the model is built, and check the accuracy of the first-order approximation when the behavioral and technical assumptions are satisfied, under a wide range of parameters.

We illustrate such "condition seeking" exercise on the Bass model.[2] Recall that the Bass model may be viewed as a special case of the proposed class of aggregate models, under the following behavioral assumptions: L = 1, and a_τ is constant for all τ and equal to a. Recall also that

$$p^{Bass} = \frac{\sum_{i=1}^{m} p_i}{m}$$

and

$$q^{Bass} = \frac{\sum_{i=1}^{m} ties_i \cdot a \cdot q_i}{m}.$$

We start by testing the accuracy of the Bass model when both behavioral and technical assumptions are satisfied. Next, we test the impact of violating the technical assumption that ties between agents are randomly reformed at each period.

Accuracy of the Bass model when its technical and behavioral assumptions are satisfied

Throughout this section we assume that both the technical and behavioral assumptions of the Bass model are satisfied. We first explore the homogenous case in which $p_i = p$, $q_i = q$, and $ties_i = ties$ for all i. We find the Bass model to be accurate for a wide range of values of p^{Bass} and q^{Bass}. See Technical Appendix C at http://www.msi.org/techapp/08-103 for details.

We focus next on the case in which p_i and q_i are heterogeneous across consumers, and explore whether the Bass model based on the *average* values of p_i and q_i continues to be accurate when there is heterogeneity in these parameters.

For simplicity we assume that $ties_i$ is constant across consumers and equal to 10,[3] and that a is equal to 1. We explore various distributions for p_i and q_i using a 2 (unimodal versus bimodal distribution) x 3 (heterogeneity on p_i and q_i versus heterogeneity on q_i only versus heterogeneity on p_i only) experimental design. Table 2 describes the distributions of p_i and q_i in each condition. We parameterize the distributions to capture fairly large amounts of heterogeneity. In all cases, the aggregate first order approximation of the diffusion process is the Bass model with p^{Bass}= .005, and q^{Bass}= .3, which are typical values (see Lilien, Rangaswamy, and Van den Bulte 2000; Sultan, Farley, and Lehmann 1990). For each condition, we simulate a "true" diffusion process for 1,000 independent populations of m = 500 agents. We report the results in Figure 2. We see that in all cases the Bass model remains within the 95% confidence intervals defined by the agent-based simulations of the "true" process. Therefore, the Bass model seems to remain a good approximation of the diffusion process even when there is significant heterogeneity in p_i and/or q_i across consumers.

Table 2

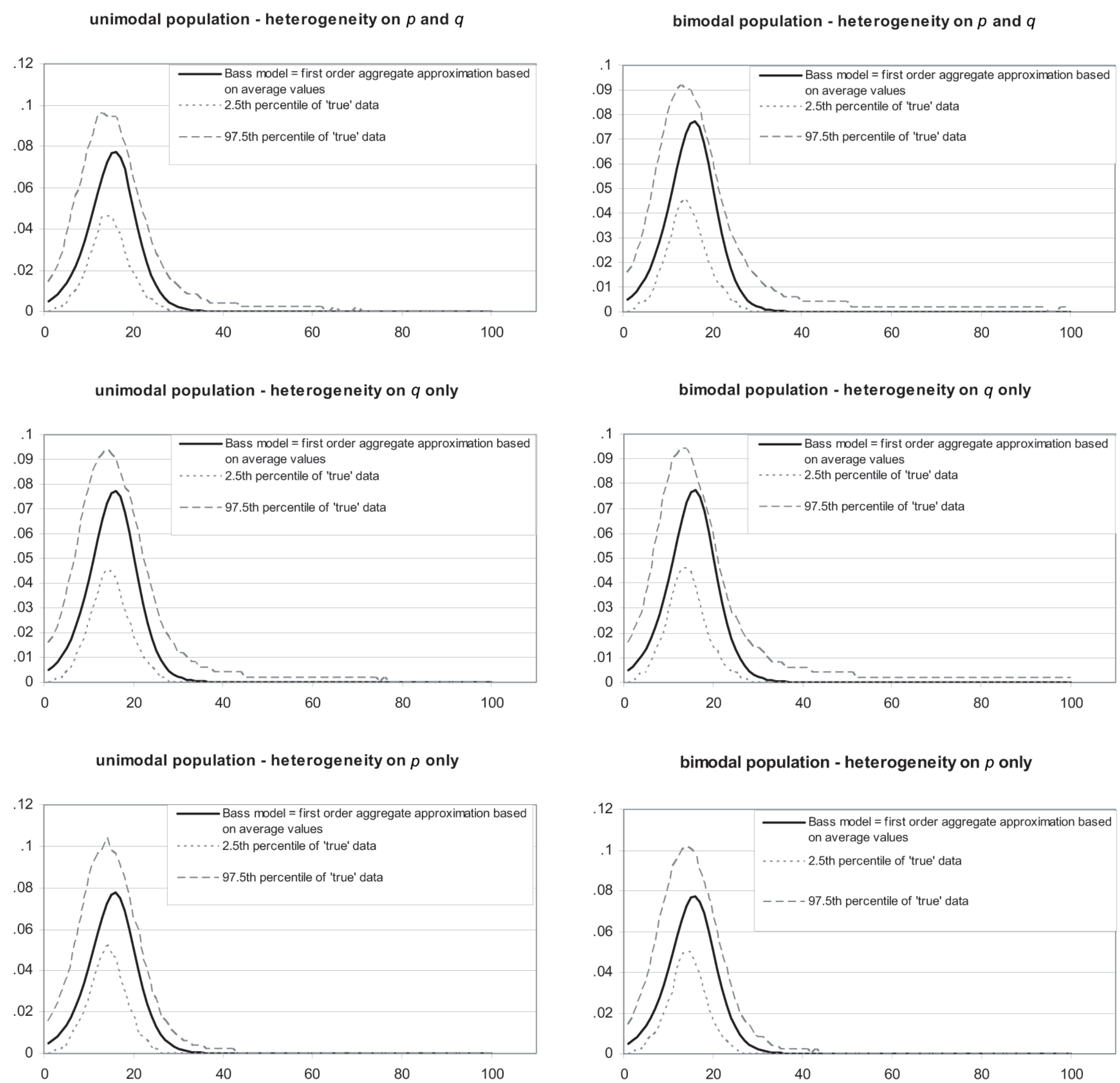
Distribution of p_i and q_i across Consumers

	Unimodal	Bimodal
Heterogeneity on p_i and q_i	$p_i \sim N(.005,.002)$	$p_i \sim .5 \times N(.004,.002) + .5 \times N(.006,.002)$
	$q_i \sim N(.03,.01)$	$q_i \sim .5 \times N(.02,.01) + .5 \times N(.04,.01)$
Heterogeneity on q_i only	$p_i = .005$	$p_i = .005$
	$q_i \sim N(.03,.01)$	$q_i \sim .5 \times N(.02,.01) + .5 \times N(.04,.01)$
Heterogeneity on p_i only	$p_i \sim N(.005,.002)$	$p_i \sim .5 \times N(.004,.002) + .5 \times N(.006,.002)$
	$q_i = .03$	$q_i = .03$

Note: All distributions were truncated such that $p_i \in [0;.01]$ and $q_i \in [0;.06]$.

Figure 2

Accuracy of Bass Model When All Assumptions Are Satisfied—Heterogeneity in p_i and q_i



Accuracy of the Bass model when its technical assumption is violated (fixed ties between consumers)

We now test the impact of violating the technical assumption that the ties between agents are randomly reformed at each period. Situations in which this assumption would be violated include networks in which consumers tend to interact with the same set of other consumers at every period. We simulate 1,000 independent populations of m = 500 agents, and simulate the "true" diffusion process assuming that a = 1 and that p_i is constant and equal to p = .005. We assume that $ties_i$ is constant and equal to $ties$, and try the following values for $ties$: 1, 2, 3, 4, 5, 10, 20. We assume that q_i is constant across consumers and equal to q, and for each value of the parameter $ties$ select q to hold the equivalent q^{Bass} constant to the typical value of .3 (i.e., $q = q^{Bass}/ties \cdot a$). For each population, we draw the ties between agents randomly only once before the first period, and hold them constant throughout the process. The first-order aggregate approximation obtained under the now incorrect assumption that ties are reformed at each period is the Bass model with p^{Bass} = .005 and q^{Bass} =.3.

Figure 3 reports the 95% confidence interval of the "true" diffusion process, as well as the diffusion process suggested by the Bass model. The results suggest that the Bass model remains accurate when the ties are static *only if the number of ties per agent is large enough*. When the ties are static and the number of ties per agent is small (in our case, less than 5), the true effect of social interactions is much more limited than assumed by the Bass model, which largely overestimates the speed of diffusion. In such situations, an adopter is able to influence only a very small number (*ties*) of potential adopters throughout the innovation process, and this influence stops once all of his or her ties have adopted. On the other hand, the Bass model assumes that the ties are reformed at each period, thereby allowing early adopters to influence a greater number of potential adopters (larger than *ties*) throughout the process.

It is possible that although the Bass model with $p^{Bass} \equiv p$ and with $q^{Bass} \equiv ties \cdot a \cdot q$ provides an inaccurate description of the diffusion process in these cases, different sets of p^{Bass} and q^{Bass} would make correct predictions. Figure 4 plots the Bass model obtained by assuming the correct m and by estimating p^{Bass} and q^{Bass} based on the "true" penetration data (averaged across the 1,000 simulated populations) from the first 10 and 20 periods, for $ties$ = 1, 2 ,3, and 4. We see that although in some cases the Bass model based on the first 20 periods makes correct predictions (when $ties$ = 3 or 4), the predictions are still inaccurate in the other cases.

Our analysis therefore suggests that it is advisable *not* to use the Bass model in situations in which the density of the consumer network is very small and the connections between consumers are constant over time. In such cases, the Bass model may be replaced with a disaggregate agent-based model.

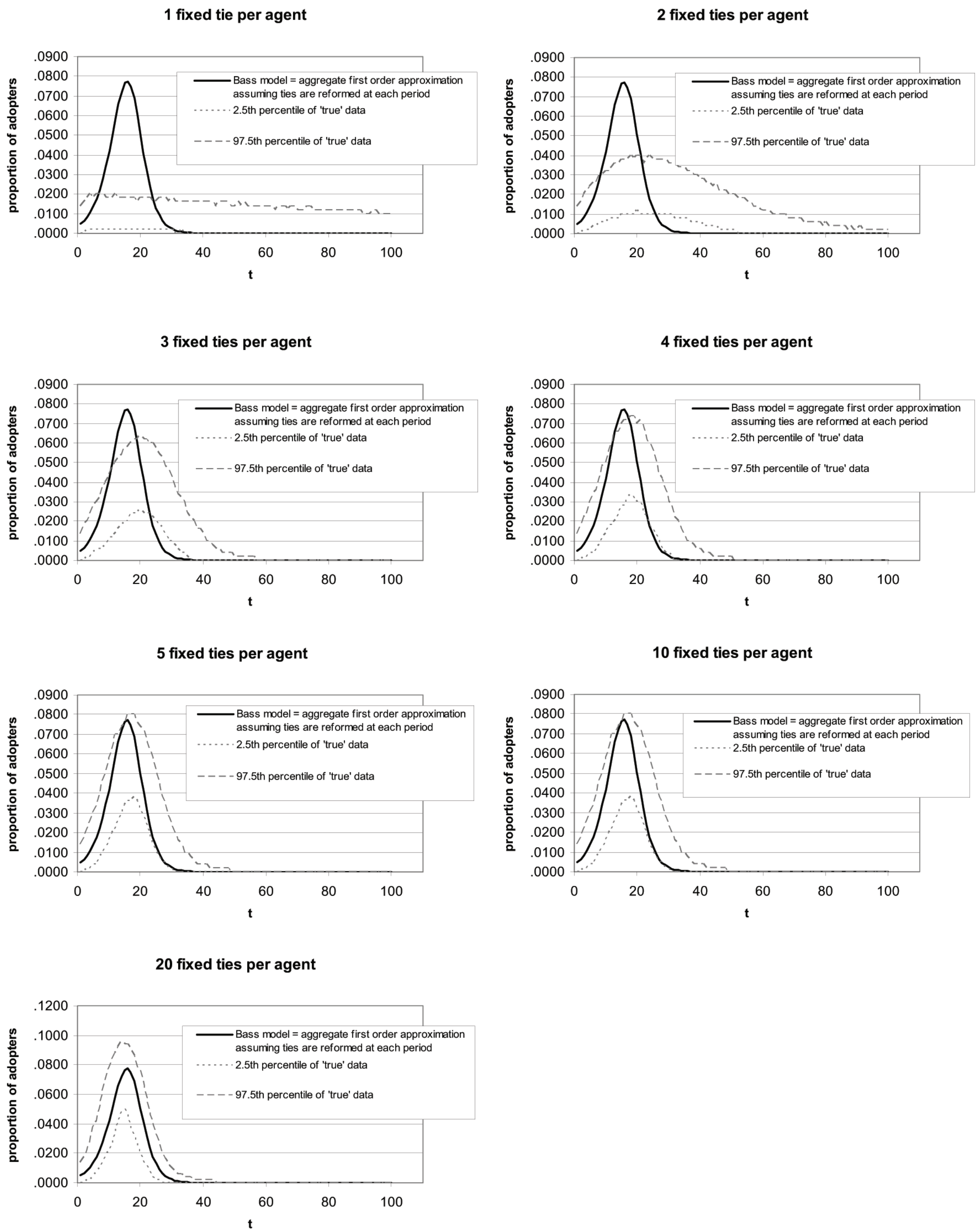
It is important to note that these recommendations apply only to the Bass model. Alternative models developed using the proposed approach may be more or less robust to violations of the technical assumption that ties between consumers are reformed randomly at each period. Therefore, we recommend performing a similar condition-seeking exercise on any aggregate model constructed using the approach proposed in this paper.

A Managerial Application

In this section we illustrate how our approach enabled a major consumer packaged goods manufacturer to develop a theory-driven aggregate diffusion model and calibrate this model shortly after launch using survey data. The company was interested in the penetration of a new oral care product. For confidentiality reasons, we will refer to this new product as PROD.

Figure 3

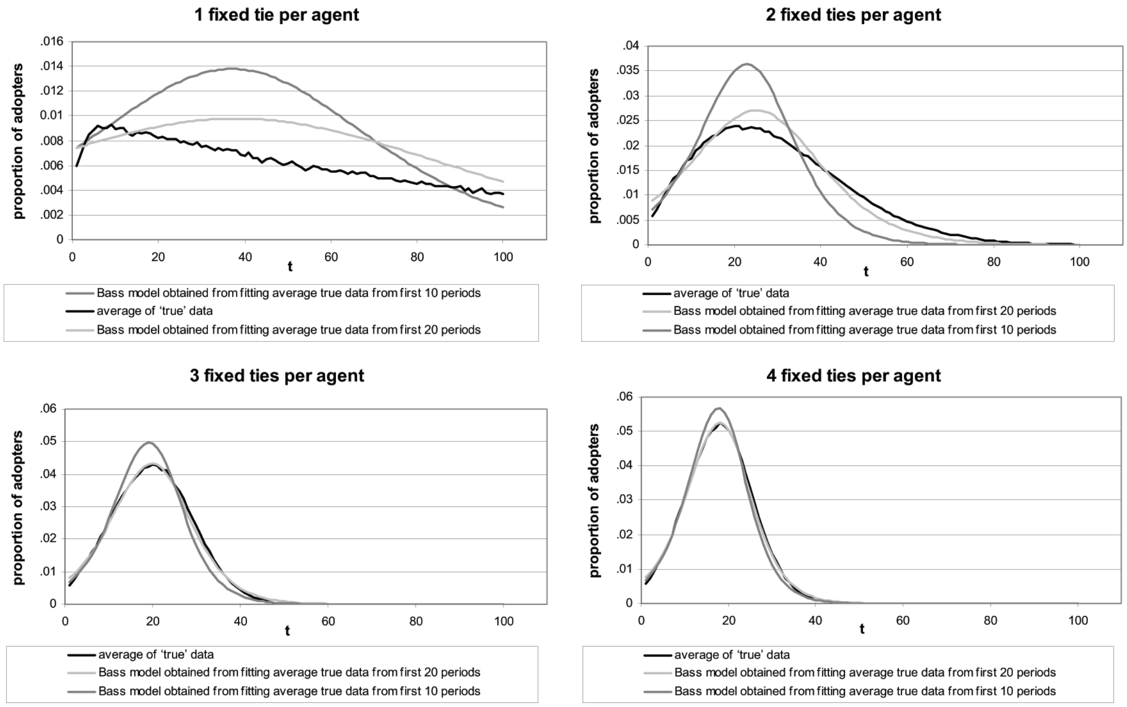
Accuracy of Bass Model When Ties Between Agents Are Static



Diffusion model

Based on discussions with the managers, we determined that the strongest form of social interactions for that category was recommendations, and that recommendations from dental professionals (i.e., dentists and hygienists) should be treated differently from recommendations from other consumers (i.e., we assumed

Figure 4
Ties Between Agents Are Static–Bass Model Obtained from Fitting Penetration Data



that $L = 2$). We also determined that adopters were most likely to recommend the product shortly after adoption. Therefore, we assumed that the adoption probability was given by:

$$P(adopt_{i,(t+1)} \mid \text{not adopted yet}) = [1 - (1-p)(1-q^{consumers})^{n_{it}^{consumers}} \cdot (1-q^{pro})^{n_{it}^{pro}}]$$

where:

- $n_{it}^{consumers}$ is the number of recommendations received by agent i from other consumers in period t,
- n_{it}^{pro} is the number of recommendations received by agent i from dental professionals in period t,
- $q^{consumers}$ is the probability of adopting based on one consumer recommendation,
- q^{pro} is the probability of adopting based on one professional recommendation.

We assumed that:

$$n_{it}^{consumers} \sim B\left(ties_i^{consumers}, a^{consumers}\frac{(S_t - S_{t-1})}{m-1}\right)$$

and $n_{it}^{pro} \sim B(ties_i^{pro}, a^{pro})$, where $a^{consumers}$ is the probability that consumer j who has adopted PROD recommends it to agent i with whom he or she is connected,[4] and a^{pro} is the probability that a dental professional connected to agent i recommends PROD to this patient. Our aggregate weekly diffusion model becomes:

$$\frac{S_{t+1}}{m} = \frac{S_t}{m} + \Bigg(p + \frac{\sum_{i=1}^{m} ties_i^{consumers}}{m} q^{consumers} \cdot a^{consumers}\frac{S_t - S_{t-1}}{m} + \frac{\sum_{i=1}^{m} ties_i^{pro}}{m} \cdot p_{recom}^{pro} \cdot a^{pro}\Bigg) \cdot \left(1 - \frac{S_t}{m}\right) \qquad (4)$$

We can rewrite Equation 4 as:

$$\frac{S_{t+1}}{m} = \frac{S_t}{m} + \left(p + \alpha \cdot q^{consumers} \cdot \frac{S_t - S_{t-1}}{m} + \beta \cdot q^{pro}\right) \cdot \left(1 - \frac{S_t}{m}\right)$$

where

$$\alpha = \frac{\sum_{i=1}^{m} ties_i^{consumers} \cdot a^{consumers}}{m}$$

is the average number of recommendations made by an adopter, and

$$\beta = \frac{\sum_{i=1}^{m} ties_i^{pro} \cdot a^{pro}}{m}$$

is the average number of recommendations received from dental professionals per week. Note that calibrating the model requires only the estimation of $q^{consumers}$, q^{pro}, α, and β, and that it is not necessary to estimate $a^{consumers}$, a^{pro}, $ties_i^{consumers}$, and $ties_i^{pro}$. Note also that $q^{consumers}$, q^{pro}, α, and β are well-defined variables that may be measured using survey data. We provide a summary of the model, its variables, and the calibration procedure in Appendix A.

Survey

In order to estimate the relevant parameters, we administered an online survey through a professional market research company, 21 weeks after launch. The respondents were 1,239 members of a representative panel of consumers. Our survey started with a set of screening questions designed to ensure the quality of the responses. We presented respondents with a list of oral care brands and asked them to indicate which ones they were aware of. The list included a set of fictitious brands, and we removed from the analysis all respondents who indicated that they were aware of at least one fictitious brand. After this screening, we were left with I = 584 respondents. The very high proportion of respondents screened out, despite the fact that the survey was performed by a professional market research company, suggests that great care should be taken to ensure the quality of online data.

The heart of the survey consisted of the following three questions:

- “Have you purchased PROD before?”
- “How many people have recommended PROD to you?”
- “How many, if any, of these people were dental professionals?”

Let us denote by y_i the binary variable equal to 1 if consumer i has adopted PROD in the first 21 weeks after launch, and by $N_i^{consumers}$ and N_i^{pro} the number of recommendations received by consumer i in the first 21 weeks after launch, from other consumers and from dental professionals, respectively ($N_i^{consumers} = \sum_{t=1}^{21} n_{it}^{consumers}$ and $N_i^{pro} = \sum_{t=1}^{21} n_{it}^{pro}$).

Calibration of the model

Estimation of p, $q^{consumers}$, and q^{pro}. We estimated p, $q^{consumers}$, and q^{pro} using maximum likelihood. The probability that consumer i has adopted within the first 21 weeks is given by:

$$w_i = 1 - (1-p)^{21} \cdot (1-q^{consumers})^{N_i^{consumers}} \cdot (1-q^{pro})^{N_i^{pro}}$$

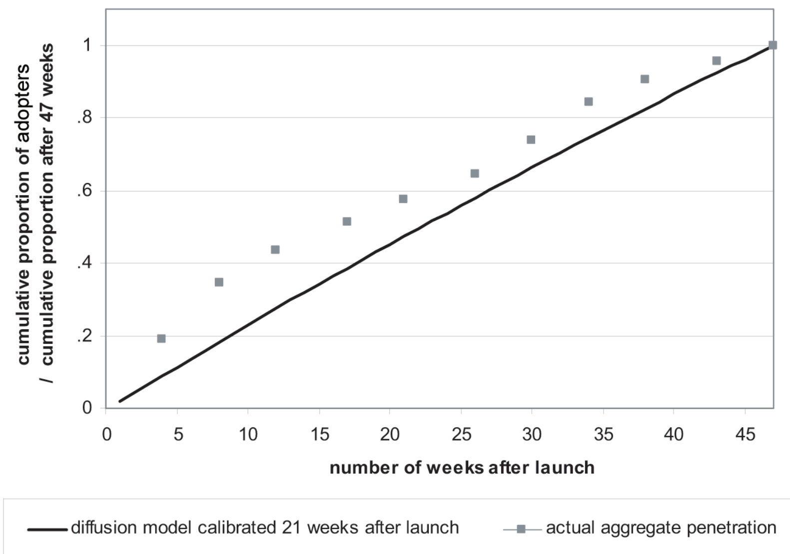
The log-likelihood of the set of y_i variables is therefore

$$\sum_{i=1}^{I} \log(w_i)^{y_i} \cdot \log(1-w_i)^{1-y_i}.$$

Maximizing this expression yields the following estimates: $\hat{p}$ = .0038, $\hat{q}^{consumers}$ = .34, and $\hat{q}^{pro}$ = .49. The managers felt that these estimates had good face validity and were consistent with other estimates obtained from earlier studies using different methods. In particular, the values of $\hat{q}^{consumers}$ and $\hat{q}^{pro}$ suggest that recommendations are influential in this category, with professional recommendations having a higher probability of leading to adoption compared to non-professional recommendations.

Estimation of α and β. The parameter α represents the average number of recommendations made per adopter. Let S_{21}/m be the proportion of consumers who have adopted PROD by week 21. These consumers have made a total of $S_{21} \cdot \alpha$ recommendations (in expectation). Therefore, the average number of recommendations received per consumer

Figure 5
Field Application: Actual and Predicted Diffusion Patterns



by week 21 is $^{S_{21}\cdot\alpha}/_{m}$, which may be approximated by

$$\frac{\sum_{i=1}^{I} N_i^{consumers}}{I}.$$

Approximating $^{S_{21}}/_{m}$ by

$$\frac{\sum_{i=1}^{I} y_i}{I}$$

gives:

$$\frac{\sum_{i=1}^{I} y_i}{I}\cdot\hat{\alpha} = \frac{\sum_{i=1}^{I} N_i^{consumers}}{I} \Rightarrow \hat{\alpha} = \frac{\sum_{i=1}^{I} N_i^{consumers}}{\sum_{i=1}^{I} y_i}.$$

We obtained an estimate of $\hat{\alpha}$ = .26, i.e., a consumer who adopts PROD recommends the product to an average of .26 other consumers.

The parameter β is equal to the average number of professional recommendations received per week per consumer, and may be estimated simply by

$$\frac{\sum_{i=1}^{I} N_i^{pro}}{21\cdot I},$$

yielding $\hat{\beta}$ = .00073. This estimate corresponds to approximately .04 recommendations per year.

Aggregate diffusion pattern

The managers were not interested in the absolute number of adopters as much as in the shape of the diffusion curve itself. Therefore they decided not to go through the exercise of determining *m*. The company shared monthly actual aggregate penetration data with us, up to 47 weeks after launch.

The very small values of $\hat{\alpha}$ and $\hat{\beta}$ suggest that although consumers trust recommendations made by other consumers ($\hat{q}^{consumers}$ = .34) and by professionals ($\hat{q}^{pro}$ = .49), adopters of PROD as well as dental professionals are very unlikely to recommend this product. As a result, the net effect of recommendations on the diffusion of this product is minimal. Figure 5 plots the shape of the cumulative diffusion curve predicted by our model as well as the actual curve based on the penetration data provided by the company. The *y* axis on the figure is equal to the proportion of adopters normalized by the predicted proportion of adopters by week 47 (for our model) or by the actual proportion of adopters by week 47 (for the actual data). We see that both the actual and predicted diffusion curves are very close to linear and dominated by *p*.

Note that fitting the Bass model on the aggregate diffusion process would give rise to a very small value of *q*, but would not offer any insight as to *why q* is small. On the other hand, our model enabled the managers to disentangle the propensity to recommend and the probability of adopting based on recommendations, and to identify specific actions that could be taken to promote the diffusion of this new product.

Conclusions and Directions for Future Research

Our approach for constructing aggregate diffusion models offers the following benefits: (1) it allows developing rich models based on explicit behavioral assumptions regarding the adoption process; (2) these models may be calibrated shortly after launch using survey data; (3) it is possible to systematically identify conditions under which these models are appropriate and conditions under which they should be replaced with alternative models.

Clearly, this research is only a first step. We believe that several areas for future research may be identified. First, although our field application allowed illustrating the approach, other applications should be performed in which social interactions play a more critical role. Second, the predictive ability of the proposed models should be tested further. Third, while we considered three specific models in this paper (including the Bass model), other models should be explored and applied. Fourth, while the class of aggregate models considered in this paper is fairly general, it may be extended further to capture additional effects such as negative word-of-mouth or complex network structures. Fifth, more sophisticated methods may be developed to calibrate the parameters of the proposed model based on survey data. Sixth, methods may be developed to calibrate the proposed models based on a *combination* of aggregate penetration data, survey data, and managerial judgment.

Acknowledgments

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Appendix A: Summary of Model Used in Managerial Application

Model:

$$\frac{S_{t+1}}{m} = \frac{S_t}{m} + \left(p + \alpha \cdot q^{consumers} \cdot \frac{S_t - S_{t-1}}{m} + \beta \cdot q^{pro}\right) \cdot \left(1 - \frac{S_t}{m}\right)$$

Model variables:

- α: average number of recommendations made by an adopter
- β: average number of recommendations received from dental professionals per week
- $q^{consumers}$: probability of adopting based on one consumer recommendation
- q^{pro}: probability of adopting based on one professional recommendation
- p: captures the effect of external forces
- ${}^{S_t}/_{m}$: the number of adopters at the end of week t

Input from survey:

- y_i: binary variable equal to 1 if consumer i has adopted the product in the first 21 weeks after launch
- $N_i^{consumers}$: number of recommendations received by consumer i from other consumers in the first 21 weeks after launch
- and N_i^{pro}: number of recommendations received by consumer i from dental professionals in the first 21 weeks after launch

Calibration of the model:

- $\hat{p}$, $\hat{q}^{consumers}$ and $\hat{q}^{pro}$ were obtained by maximizing the log-likelihood function:

  $$\sum_{i=1}^{I} \log(w_i)^{y_i} \cdot \log(1 - w_i)^{1-y_i},$$

  where $w_i = 1 - (1 - p)^{21} \cdot (1 - q^{consumers})^{N_i^{consumers}} \cdot (1 - q^{pro})^{N_i^{pro}}$

- $$\hat{\alpha} = \frac{\sum_{i=1}^{I} N_i^{consumers}}{\sum_{i=1}^{I} y_i}$$

- $$\hat{\beta} = \frac{\sum_{i=1}^{I} N_i^{pro}}{21 \cdot I}$$

Notes

1. Note that "resisting" the innovation here may be either a conscious or unconscious decision, i.e., the consumer does not have to be aware of the innovation to "resist" it.

2. The term "condition seeking" is borrowed from the philosophy of science literature (Greenwald et al. 1986).

3. Simulations available from the authors explore the case in which $ties_i$ varies across consumers. In all cases, the Bass model is found to be accurate as long as the behavioral and technical assumptions are satisfied.

4. Simulations, available from the authors, suggest that for this application the predictions were not very sensitive to the number of periods over which the recommendations are made. For simplicity, we assumed that adopters made all their recommendations in one period.

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