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The gravitational law of social interaction

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HIGHLIGHTS

- We empirically investigate the relation between distance and link probability.
- Four very different social networks are examined.
- We find that the probability decreases as the inverse of the distance squared.
- This is the exact unique distance dependence that ensures network searchability.

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ABSTRACT

While a great deal is known about the topology of social networks, there is much less agreement about the geographical structure of these networks. The fundamental question in this context is: how does the probability of a social link between two individuals depend on the physical distance between them? While it is clear that the probability decreases with the distance, various studies have found different functional forms for this dependence. The exact form of the distance dependence has crucial implications for network searchability and dynamics: Kleinberg (2000) [15] shows that the small-world property holds if the probability of a social link is a power-law function of the distance with power -2, but not with any other power. We investigate the distance dependence of link probability empirically by analyzing four very different sets of data: Facebook links, data from the electronic version of the Small-World experiment, email messages, and data from detailed personal interviews. All four datasets reveal the same empirical regularity: the probability of a social link is proportional to the inverse of the square of the distance between the two individuals, analogously to the distance dependence of the gravitational force. Thus, it seems that social networks spontaneously converge to the exact unique distance dependence that ensures the Small-World property.

1. Introduction

A great deal has been learned about the topology of networks, and social networks in particular [1–4]. Yet, the geographical structure of these networks is still not fully understood. Different studies report different functional forms of the dependence of the probability of a social link on the distance. The exact distance dependence has important implications for network searchability and for understanding, and possibly influencing, social dynamics, and it is the focus of the present study.

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Studies that have investigated the geographical structure of social networks typically focus on one particular network, and investigate it in depth. These studies report different results regarding the role of distance in social interaction. For example, in mobile phone communications the probability of a social link depends on the distance r between individuals as $1/r^2$ [5]. Similar results are obtained when analyzing mobile phone communications data at the city level [6]. In contrast, in the LiveJournal network the probability of a link is proportional to $1/r^{1.2} + \varepsilon$, where ε is a background probability that is independent of distance [7]. In email communications between employees within an HP lab the link probability was found to follow $1/r$ [8]. Similar findings are obtained in Ref. [9], where an algorithm to determine a person's location by information about her friends' locations is developed. Refs. [10,11] study location-based social networks, such as Brightkite, Foursquare, and Gowalla, and find that the probability of a social link in these networks generally depends on the distance as a power-law, with power in the range of -0.5 to -1.0. For an excellent review of these and other studies, see Ref. [12]. Do these different results stem from the different social networks analyzed? Are the differences due to the different methodologies employed? (For example, some studies use proxies, such as the average check-in location, to determine a person's "home location", while others use exact addresses or zip-codes.) Perhaps some of the differences are due to the very different distance scales involved? (In the HP email study the maximal distance is 1000 feet, or about 0.3 km, while the maximal distance in the other studies is in the order of hundreds, or even thousands, of km.) The purpose of this study is to analyze a variety of social networks within a unified framework, and to examine whether a general regularity emerges. We investigate the geographical structure of social networks by analyzing four independent and very different sets of data: Facebook links, the social links of participants in the electronic-version Small-World experiment [13], email communication, and data from in-depth personal interviews reported in Ref. [14].

2. Data and results

2.1. Facebook links

"MyPersonality" is a Facebook application that allows users to examine their personality profile. The application also allows users to share their personal data, including their home address zip-code.[1,2]

By examining all pairs of Facebook friends (i.e. pairs of linked users) in the MyPersonality database, we constructed the distribution of link distances. The dataset contains 289,432 users who provided their zip-codes. Of these, we found 531,223 linked pairs with reported zip-codes. The distance between any two linked users was calculated by taking each user's location as the "center of mass" of his zip-code area, and calculating the distance between the two locations.

The probability of a social link is estimated by dividing the *actual* number of existing links at a certain distance by the *potential* number of links at this distance (i.e. the total number of pairs, linked and unlinked, located at this distance one from the other). What is the potential number of social links at a given distance r? For a given person, the potential number of links at distance r is the total number of people located at distance r from her (i.e. in a ring of radius r around her). Under the simplistic assumption of a plane with uniform population density ρ, the number of individuals populating a thin ring of distance r and width Δr around the individual is $2\pi r \Delta r \rho$, i.e. the potential number is proportional to r. At high resolution, the population density is, of course, not uniform – cities are much denser than rural areas. However, when looking at the inter-city resolution, as we do here, the empirical potential number of links is indeed almost perfectly linear in r. Fig. 1A shows the potential number of links as a function of the distance for the MyPersonality database. For every distance range $(r, r + \Delta r)$ we calculate the total number of pairs (linked and unlinked) located at this distance one from the other. The linear fit is very good with $R = 0.996$ ($R^2 = 0.99$).

Panel 1B reports the distribution of *actual* Facebook links as a function of physical distance, on a double-logarithmic scale. The number of links is fitted very closely by a power law. The power is -1.08 with a standard error of 0.03, i.e. the observed distribution of the number of links is very closely approximated by the inverse of the distance, $1/r$, shown by the dashed line with slope -1.

Given that the actual number of links at a distance r is closely approximated by $1/r$, and that the potential number of links is proportional to r, this implies that the probability of a social link is proportional to $\frac{(1/r)}{r} = \frac{1}{r^2}$. This is directly confirmed in Panel C of Fig. 1, which shows the ratio between the actual number of links and the potential number of links directly calculated for each distance interval. The scale is double-logarithmic, and thus the linear fit implies a power-law distribution. The best-fit power is -1.98 ± 0.11. Thus, the probability of a link is fitted very well by a power-law of the distance with power -2 (shown by the dashed line):

$$p(r) \propto \frac{1}{r^2}. \tag{1}$$

[1] We thank David Stidwell, the developer of the "My Personality" application, for this anonymized data.

[2] About 5% of the users reported their zip-codes. The sample of Facebook users who voluntarily report their location is not systematically biased relative to the entire user population [9].

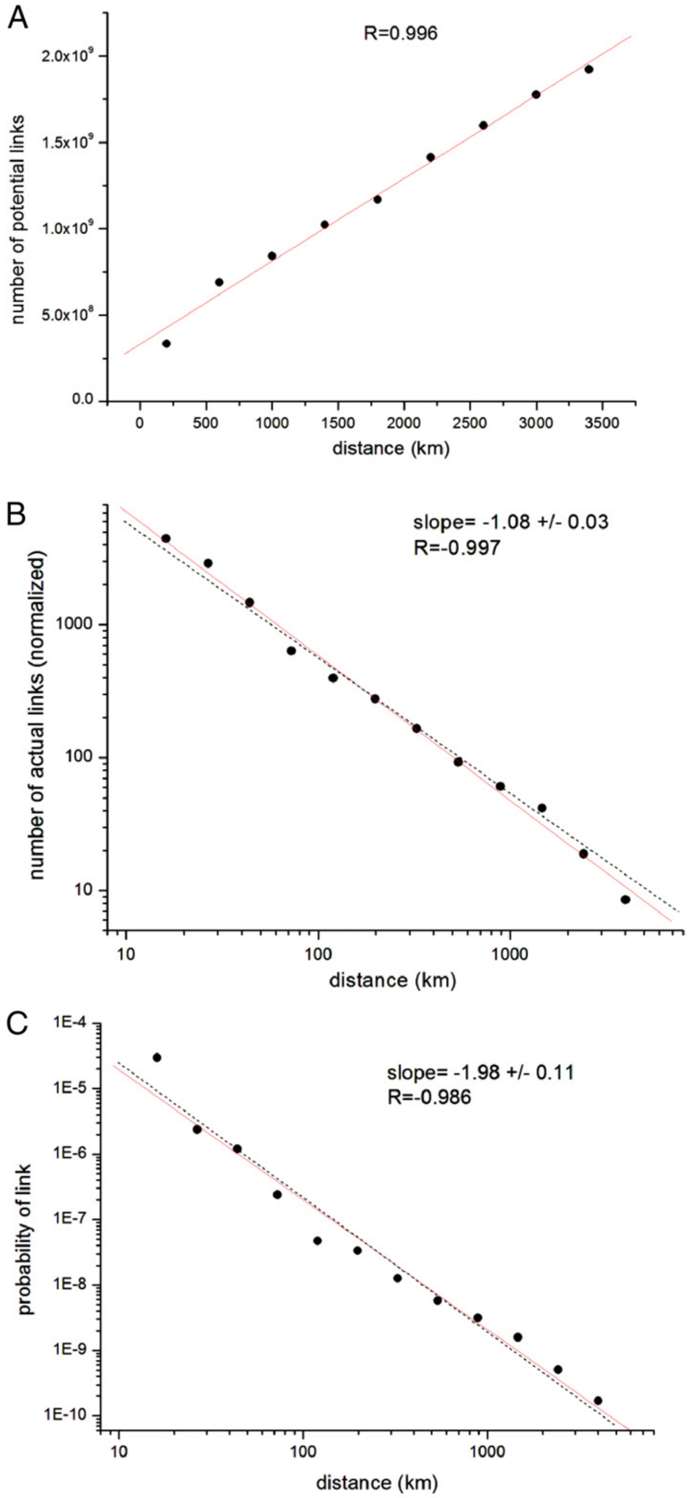

Fig. 1. A. Number of *potential* Facebook links as a function of the distance. B. Number of *actual* Facebook links (log–log scale). The distance bins are equally sized on the log scale, and the number of links in each bin is normalized to take into account the bin size. The dashed line shows the inverse-distance relation (slope of -1). C. The probability of a link as a function of the distance. The gravitational law (slope -2) is shown by the dashed line.

This dependence is exactly the distance dependence that ensures the small-world property [15,16]. The inverse square-distance dependence in Eq. (1) is also analogous to the distance dependence of the Newtonian gravitational force (and the electric force), and can therefore be called "the gravitational law of social interaction".

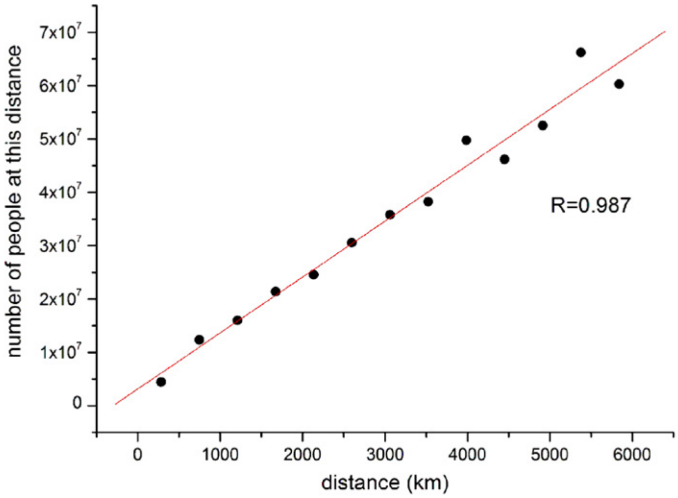

Fig. 2. Number of people at distance r from an individual. The largest 60,000 cities in the world are employed. For each city, we have its population and geographical location. Given this information, we calculate the potential number of links at each distance. For distances on the inter-city scale the number of people living at distance r from a point is almost perfectly proportional to r. At larger distances the Earth's finite size and its curvature begin to play a role.
Source: Google Geocoding http://code.google.com/intl/iw-IL/apis/maps/documentation/geocoding.

2.2. The small-world experiment

In a classic experiment, Milgram and Travers ask a number of "source" individuals to forward a letter to a designated "target" person [17,18]. Each individual is only allowed to pass-on the letter to someone s/he knows on a first-name basis. The objective is to deliver the letter to the target person in a few "steps" as possible. The 296 source individuals were either from Nebraska or from Boston, and the target person was from the Boston area. Milgram and Travers found that the average number of intermediary steps between the source person and the target person is about 6, which led to the famous phrase of "six degrees of separation", and to the understanding that we are part of a "Small-World" social network. Dodds, Muhamad and Watts [13] conduct a modern email version of the Small-World experiment on a very large scale, with over 20,000 "source" individuals and 18 "target" individuals located across the globe. They find an average degree of separation between 5 and 7, and thus extend the small-world result to the global social network. We employ the data of the Dodds, Muhamad and Watts experiment to investigate the probability of a social link as a function of the geographical distance.[3]

In the MyPersonality database we have the entire population of users, from which we calculate the potential number of links at any given distance. In the Small-World experiment, and in the email and personal interview datasets, any person on the globe constitutes a potential link. Thus, to estimate the distribution of potential links in these three cases we look at the entire world population. We take the 60,000 largest cities in the world, where for each of these cities we have its population and location (source: Google Geocoding http://code.google.com/intl/iw-IL/apis/maps/documentation/geocoding). Given this information, we calculate the number of potential social links as a function of the distance. The results are shown in Fig. 2, revealing that the number of potential links is almost perfectly proportional to the distance at the inter-city level ($R = 0.987$). This is consistent with the results for MyPersonality users (shown in Fig. 1A), and also with the naïve model of uniform population density, which evidently provides a good approximation at the inter-city level.

The small-world experiment offers an exceptional database of social links. However, the objective of reaching a particular target person introduces a systematic bias. For example, if you are asked to deliver a message to a target person who is a historian living in Moscow, you may consider sending the message to people you know in Moscow, or elsewhere in Russia. As most source individuals in the experiment are from Northern America, this may introduce an overestimation of long-range social links. Fortunately, there is a way to address this problem. Participants were asked to specify the reason they chose the recipient of their message. Possible answers were: location, profession, education, etc. For estimating the dependence of link probability on distance we took only links where the reason for choosing the recipient was *not* "location". If you select the recipient of your message because she is a historian, this should reflect your social link distribution in an unbiased way.

Fig. 3 shows the number of actual social links in the Small-World experiment for which reason $\neq$ location as a function of the distance r (double-logarithmic scale). The linear fit is almost perfect, indicative of a power law. The slope is close to -1 (-0.94 with a standard error of 0.03). The fit to the inverse distance function with slope -1 is very good (shown by the dashed line). As the actual number of links is closely approximated by $1/r$ (Fig. 3), and as the potential number of links is

3 We are very grateful to Duncan Watts for sharing this data with us.

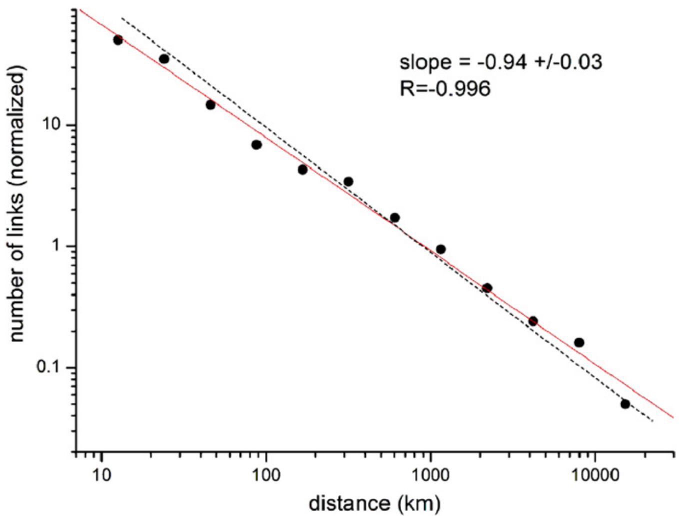

Fig. 3. Number of social links in the Small-World experiment as a function of the distance (log–log scale). The distance bins are equally sized on the log scale, and the number of links in each bin is normalized to take into account the bin size. The dashed line shows the inverse-distance relation (slope $= -1$).

proportional to r (Fig. 2), again we obtain that the probability of a social link, $(1/r)/r = 1/r^2$, follows the gravitational law, as in Eq. (1).

Note that the empirical observation that the number of links is proportional to $1/r$ is consistent with the gravitational law (Eq. (1)) whether the population density is uniform or not. For example, suppose that there are two regions A and B with very different population densities ρ_A and ρ_B. In region A, the gravitational law implies that the average number of links at distance r will be proportional to $\frac{\rho_A}{r}$. Similarly, the gravitational law implies that in region B the average number of links at distance r will be proportional to $\frac{\rho_B}{r}$. If we aggregate across the two regions, we will find that the average number of links is proportional to $\frac{w_A\rho_A+w_B\rho_B}{r}$, where w_A and w_B are the relative weights of the respective regions. Thus, the gravitational law implies that the number of links is proportional to $1/r$ even if the population density is non-uniform.

2.3. Email traffic

As it is extremely difficult to obtain detailed geographical data on email communications, due to obvious privacy issues, little is known about the volume of email communications as a function of the geographical distance between correspondents. We collected email traffic data by asking randomly selected US subjects to fill a questionnaire reporting the locations of the recipients of their last 50 email messages, and their own city of residence. A reward of $2000 was given to one randomly selected respondent for filling the questionnaire. Overall, we collected data for 4455 email messages. This number is unfortunately much smaller than the number of links we were able to obtain in the first two datasets. Thus, we can only make weaker inference about the distance dependence in this case. However, the inverse distance dependence describes the empirical number of links quite well also in this case.

Fig. 4 shows the distance dependence for the email data. Panel A reports the number of email links as a function of the distance. As the geographic location is given only at the city level of detail, the resolution of the email data is less detailed than the Facebook data (which is given at the zip-code level). Out of the 4455 messages, 1818 (41%) were sent within the same city, yielding a distance measure of zero. This is consistent with the findings in Ref. [14], showing that a large proportion of social interactions are local. Obviously, this low resolution limits our ability to characterize the density function, especially for short distances. Panel A reveals that the number of links decreases approximately as $1/r$ (see also the discussion of Panels B and C below). This implies, as before, that the *probability* of a link decreases as $1/r^2$. Note that in Panel A the scale is not logarithmic, and equally-sized bins are employed. There are not enough data points to analyze the distribution on a log–log scale, as in Figs. 2 and 3. Panel B provides a more detailed picture by presenting the cumulative distribution. The cumulative distribution for a truncated $1/r$ density function is the log function. Thus, the $1/r$ density implies a linear relation between the cumulative link distribution and $\log(r)$. The solid line describes the empirical cumulative distribution. The dashed line shows the theoretical prediction of the $1/r$ law. As can be seen in the figure, there are some deviations from the straight line, perhaps because of the lower city-level resolution of the data, but the fit is still quite reasonable: $R^2 = 0.979$.

While the density and cumulative distributions provide a good picture for most distances, for large distances the number of observations is insufficient to obtain clear results using this method, because we have only 4455 links in the email dataset. Therefore, we also employ the rank-distance method [19], which focuses on the large-distance links. In this method, shown in Panel C, links are ranked from the one with the greatest distance (rank $n = 1$) to the one with the lowest distance. The

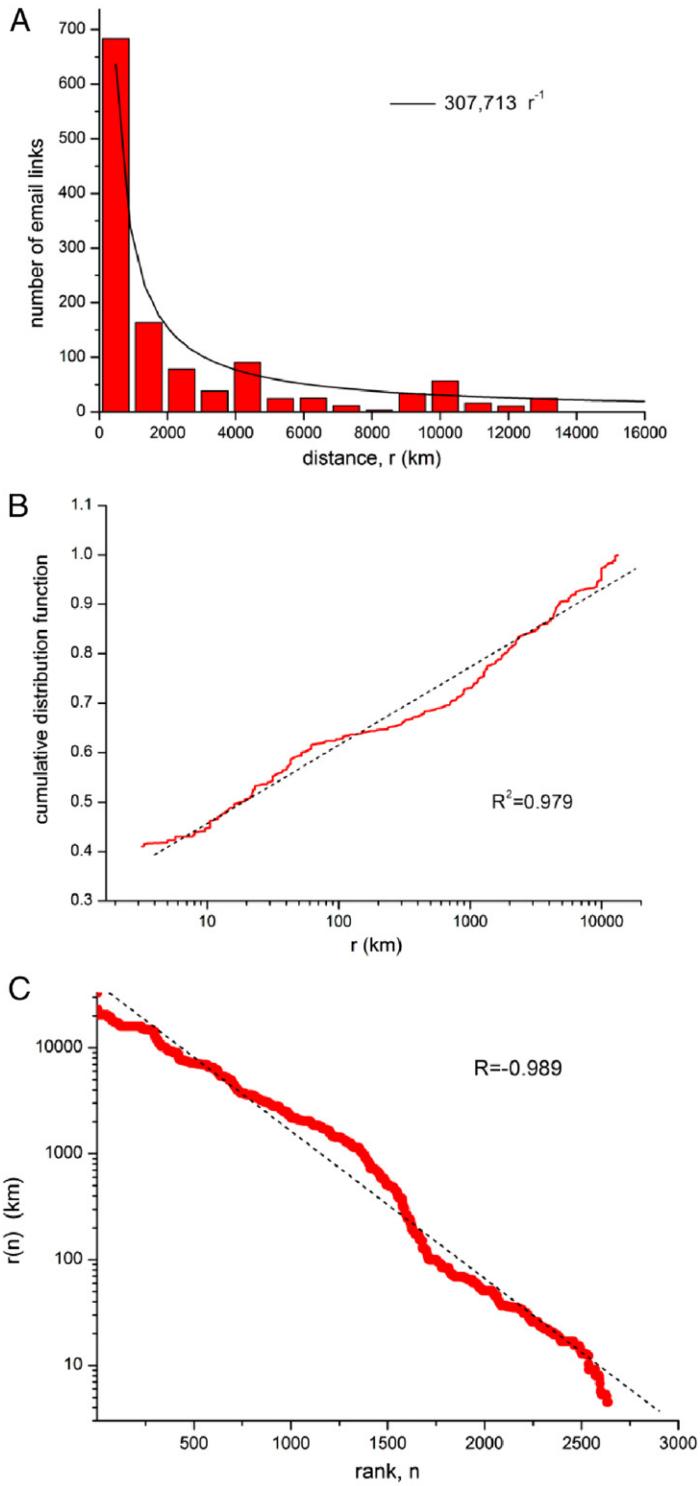

Fig. 4. The distribution of links in the email dataset. The geographic distance between correspondents was calculated for 4455 email messages originating in the USA. Panel A shows the number of email links as a function of distance. The empirical distribution is given by the bars, and the $1/r$ distribution is given by the solid line. Panel B shows the empirical cumulative distribution (solid) and the theoretical linear prediction (on a semi-logarithmic scale). Panel C provides the relationship between rank, n, and the corresponding distance $r(n)$. A linear relation between rank and distance on a semi-logarithmic scale implies the $1/r$ distribution (see footnote 4). The straight line shows the best linear fit to the empirical data, with $R = -0.989$. Note that we have only 2637 distance observations, as 1818 (41%) of the messages were sent within the same city.

$1/r$ density implies a linear relation between $\log(r)$ and rank.[4] Panel C of Fig. 4 presents the distance $r(n)$ as a function of rank, on a semi-logarithmic scale. The correlation we obtain between $\log(r(n))$and n is $R = -0.989$. Thus, while the email dataset is much smaller than the Facebook and Small-World experiment datasets, it too is consistent with the number of social links being inversely proportional to the physical distance r. Given the potential number of links which is proportional to r (Fig. 2), we obtain that the probability of a social link is proportional to $1/r^2$, again in accordance with the gravitational law of social interaction.

2.4. Personal interviews

Mok, Wellman, and Carrasco [14] investigate the role of distance in different forms of inter-personal communications, before and after the internet revolution. They conduct in-depth personal interviews with 86 individuals randomly drawn from the East York (Toronto) area. The interviewees are asked to identify individuals (or "alters") with whom they have active social ties, their locations, and their means of communications with them. Fig. 1 in Mok et al. describes the number of social ties as a function of the distance. While it is clear from this figure that the number of ties generally decreases with the distance, the focus in Mok et al. is not on the exact functional form of this distance dependence. Here we employ the data reported in Ref. [14] to examine the gravitational law hypothesis. Here too, the number of links is not large enough to allow analysis on the double-logarithmic scale as shown in Figs. 1 and 3, and we perform analysis parallel to the analysis in Fig. 4.

Fig. 5A reports the number of social ties as a function of the distance, as revealed in the personal interviews. The empirical data is shown by the histogram, while the best-fit inverse distance relation is given by the solid line. While there is general agreement between the theoretical prediction and the empirical data, the cumulative distribution in Fig. 5B, which is the analog of Fig. 4B, provides a more informative picture. The fit to the linear relation in the semi-logarithmic scale is very good ($R^2 = 0.988$), implying that the number of links decreases approximately as $1/r$. The observation that the actual number of links is proportional to $1/r$, and that the potential number of links is proportional to r (see Fig. 2), together imply, once again, that the probability of a link is proportional to $1/r^2$. Unfortunately, we were not able to obtain the list of all ties (only the aggregate numbers as a function of the distance), so we cannot perform the rank-distance analysis parallel to the analysis in Fig. 4C. Still, it is very encouraging to find confirmation for the gravitational law of social interaction in data that was collected by means very different than the typical collection methods that rely on records of electronic communication.

3. Discussion

All four datasets reveal the same regularity: the probability that two individuals located at distance r one from another are socially linked is proportional to $\frac{1}{r^2}$. This relation is analogous to the Newtonian gravitational law: the probability of a social link (the force) between two individuals (bodies of mass) is proportional to the inverse of the distance squared.

We can generalize the "gravitational law" (1) by relaxing the assumption of identical individuals, and allowing individuals to have different susceptibilities (or unconditional probabilities) of being linked. If we denote the "friendliness" of individual i by a parameter $0 \leqslant m_i \leqslant 1$, where $m_i = 1$ is the maximal friendliness and $m_i = 0$ is the minimal friendliness, we can generalize Eq. (1) to state that the conditional probability of two individuals i and j at distance r from each other being socially linked is:

$$p(r) = \frac{Gm_i m_j}{r^2}, \tag{2}$$

where G is a constant. This extension is consistent with network models in which different individuals have a very different number of social links, such as the scale-free network model with a power-law degree distribution [1,20,21].

Several studies [7,22,23] suggest that the probability of a social link between individual i and individual j (or the probability of a trip from place i to place j) is inversely proportional to the rank of individual j from the perspective of individual i, where this rank is defined as the number of people geographically closer to i than j. Note that in an area with uniform population density ρ this number is $\rho\pi r^2$, where r is the distance between the two individuals. Thus, if the link probability is inversely proportional to the rank, it is proportional to $\frac{1}{\rho\pi r^2}$, consistent with the gravitational law. For areas with different densities, the proportionality factor will be different. When aggregating across regions with different densities, as we do here, one would still obtain the gravitational law, but with a proportionality factor which is the weighted average of the proportionality factors in the different regions.

The gravitational law found here is consistent with the findings for mobile phone communications [5,6], but it is at odds with the findings regarding email communications within an HP lab [8], and those regarding the LiveJournal network [7]. How can these findings be reconciled? Ref. [8] shows that the intra-lab communications are based primarily

4 The linear relationship $\log(r(n)) = A - Bn$ implies that the number of links exceeding distance r is $n(r) = \frac{A}{B} - \frac{\log(r)}{B}$. If we denote the total number of links by N, the proportion of the distances greater than r is $n(r)/N$, and therefore the cumulative probability function $F(r)$ is $1 - n(r)/N$: $F(r) = 1 - \frac{A}{BN} + \frac{\log(r)}{BN}$. The probability density function is the derivative of $F(r)$ with respect to r, i.e. $f(r) = \frac{1}{BNr}$. Thus, a linear relationship between $\log(r(n))$ and n implies the $1/r$ density.

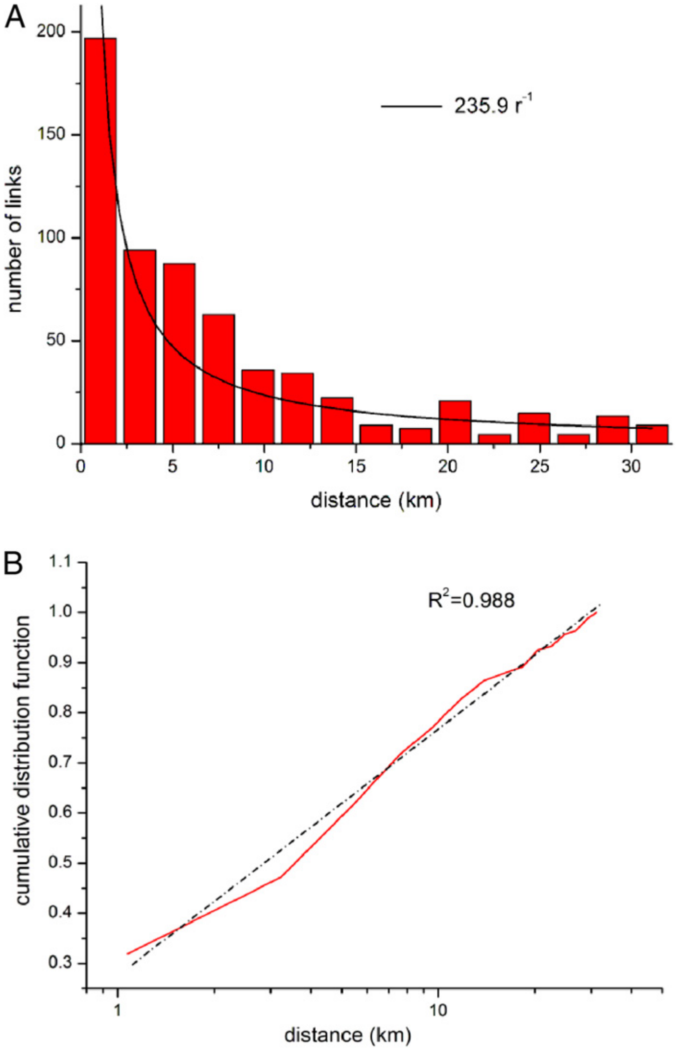

Fig. 5. The distribution of social links in the personal interviews dataset in Ref. [14]. Panel A shows the density of the distribution of email messages as a function of distance. The empirical distribution of social links is given by the histogram, and the theoretical $1/r$ distribution consistent with the gravitational law is given by the solid line. Panel B shows the empirical cumulative distribution (solid) and the (dashed) theoretical linear prediction (semi-logarithmic scale). The linear approximation is very good ($R^2 = 0.988$), implying that the number of links decreases as $1/r$ and thus the probability of a link decreases as $1/r^2$.

on the hierarchical structure of departments. Thus, these findings regarding distance dependence may reflect the distance dependence of the departmental structure rather than the distance dependence of social interaction per se. The LiveJournal data in Ref. [7] may involve a strong thematic component that is not related to distance. For example, followers of a Justin Bieber fan blog may be unrelated geographically. While this effect is addressed by the background probability ε, it may also systematically influence the exponent, and may lead to an underestimation of the role of distance in social interaction.

It is interesting, and perhaps surprising, that physical distance plays such an important role in social interaction, even in the internet era, when one can communicate easily with almost anyone else across the globe. One possible interpretation is that social links tend to be initially formed in face-to-face contacts, and that geography is still a very important factor influencing human mobility and the probability of such contacts [24–30].

The gravitational law of social interaction provides a quantitative framework for analyzing the spread of ideas/norms/fads/viruses across the social network. It is also closely related to the Small-World property. This property requires two distinct elements: (i) that short chains exist between any two individuals, and (ii) that individuals are able to identify these chains by using only local information, i.e. with information only about their immediate friends, but not about the entire structure of the network. Kleinberg [15,16] shows that both of these elements hold if the probability of a social link between two individuals located at distance r one from the other follows a power law, $p(r) \propto r^{-\alpha}$ with power $\alpha = 2$, but not with any other value of α. The empirical finding of the gravitational law implies that social networks tend to self-organize in the very special and unique way that makes the network searchable.

Is this a coincidence? What is the reason for the gravitational law of social interaction? One possible explanation, based on an informational framework, is suggested in Ref. [31]. Another possible explanation, based on a dynamic model of link formation by triadic closure, is suggested in Ref. [32]. Experimentally testing these, and possibly other, explanations for the gravitational law of social interaction, and developing the implications of this law for network dynamics, seem as challenging yet promising paths to follow.

Acknowledgments

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