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Zooming In: Self-Emergence of Movements in New Product Growth

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In this paper, we propose an individual-level approach to diffusion and growth models. By *zooming in*, we refer to the unit of analysis, which is a single consumer (instead of segments or markets) and the use of granular sales data (daily) instead of smoothed (e.g., annual) data as is more commonly used in the literature. By analyzing the high volatility of daily data, we show how changes in sales patterns can self-emerge as a direct consequence of the stochastic nature of the process. Our contention is that the fluctuations observed in more granular data are not noise, but rather consist of accurate measurement and contain valuable information. By stepping into the noise-like data and treating it as information, we generated better short-term predictions even at very early stages of the penetration process. Using a Kalman-Filter-based tracker, we demonstrate how movements can be traced and how predictions can be significantly improved. We propose that for such tasks, daily data with high volatility offer more insights than do smoothed annual data.

Key words: growth process; new product; penetration; sales movements; takeoff; diffusion; agent base modeling; forecasting; adoption; innovation; social networks
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1. Introduction

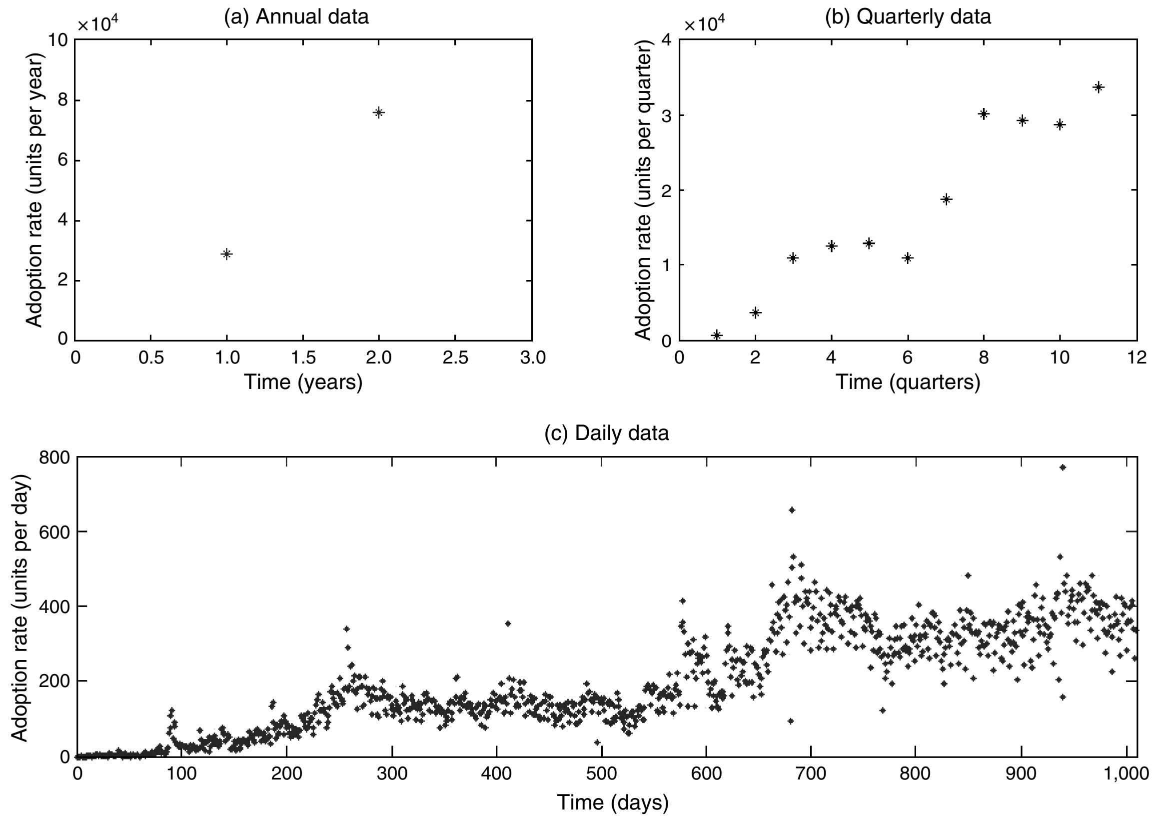
Monthly or weekly sales curves are often characterized by movements (trends), and changes in the curve with what seems to be loud *noise*. Such noise in sales or penetration data is typically handled through data smoothing and larger time frames of analysis (i.e., quarters or years). The term noise is often used (in most fields) to reflect measurement tool error. As measurement tools become increasingly accurate, and information technology developments enable better measurement, we can posit that what we see in granular data is not noise, but rather the true face of growth. One of the main claims of this research is that volatility, perhaps surprisingly, contains valuable information. Accordingly, we propose to *step inside the noise* and use granular data instead of the more commonly used smoothed quarterly or annual data (see Mahajan et al. 2000; see also Putsis 1996 on how the data frequency influences the estimations; monthly data add little on quarterly). Taking this approach, fluctuations can be used as information instead of noise that has to be cleaned, enabling more accurate postlaunch predictions at much earlier stages (i.e., right after introduction).

To illustrate our claim, consider Figure 1, which presents the adoption rate (in units) of an e-mail software[1] on an annual scale (Figure 1(a)), quarterly scale (Figure 1(b)), and daily scale (Figure 1(c)) during its first 1,010 days of penetration into the Indian market. Naturally, the annual penetration data (Figure 1(a)) produce just two data points, revealing very limited information on the temporal evolution of the product's penetration. When the scale resolution is increased to quarterly penetration (Figure 1(b)), the primary tendency of growth is more clearly shown, wherein the apparent oscillations can be interpreted merely as fluctuations of the adoption rates. However, as can be seen in Figure 1(c), there are movements (changing trends) in daily data that can hardly be seen in the more aggregated data as in Figures 1(a) or 1(b).

It is not obvious at which time resolution the data are best analyzed. The annual data (Figure 1(a)) offer only two data points, providing scant insight, while the daily data (Figure 1(c)) involve high volatility that may appear to be noise. A possible compromise might be waiting several years or using quarterly data that

[1] The software is an advanced e-mail program ("IncrediMail") that contains a useful toolbar, an interface that allows changes to the outline of the e-mail, as well as programming and customization of various alerts. The software itself was introduced in 2000 and since then has become a recognized e-mail interface all over the world.

Figure 1 **Actual Penetration of a Software Product in (a) Annual Scale (b) Quarterly Scale, and (c) Daily Scale**



offer enough data points to make forecasts based on smoothing and fitting, yet are less noisy than the daily data. But is this our best option?

In this paper, we show that the patterns observed in Figure 1(c) are actually an inherent component of the diffusion process of new products, and using granular data (rather than smoothing it) offers a better way to make predictions, as well as very early stages and more accurate short-term forecasting.

The structure of the rest of the paper is as follows: First, we discuss the motivation for using the proposed approach in the case of new product adoption and growth. Next we present a framework for synthesizing both individual and aggregate levels in the most general case into a universal modeling platform. In §3, we show how penetration movements can be a result of an intrinsic stochasticity of the process, and in §4, we demonstrate how the proposed approach enables us to perform better short-term penetration forecasting (e.g., for the next quarter or two) as early as at the beginning of the adoption process, using actual penetration data. We conclude with a discussion in §5.

1.1. Background

Postlaunch evaluation of new product performance based on sales data is an important tool frequently used by firms to plan ahead for production and marketing resource allocation. In their review of the research on innovation, Hauser et al. (2006) covered research challenges in growth of new products, emphasizing, among other things, the need for generalizations of the S-shaped curve and turning points, as well as determining how network effects influence diffusion. This paper follows their recommendations by focusing on the effects of social network structure on the penetration stage.

One common problem shared by most approaches used in this area is that the early stages of product introduction do not necessarily fit into the *diffusion of innovation* framework. Reviews of diffusion models (e.g., Parker 1994, Mahajan et al. 2000, Golder and Tellis 1997, Tellis et al. 2003) find little use of growth models around takeoff. This state of affairs might be explained in several ways. First, modeling the initial phase of the postlaunch process may have relatively limited efficacy; i.e., only a small number of data points exist before takeoff. Indeed, Kohli et al. (1999) argued that the Bass model, while excellent at backcasting, is unreliable on its own for early life-cycle forecasting. Related claims point to the lack of data reliability prior to the completion of several periods of sales and the emergence of sales curve stabilization (Heeler and Hustad 1980, Parker 1994). Our proposed approach alleviates some of these concerns.

Furthermore, in the presence of volatility, which is quite common during introduction stages, the search for explicit solutions for nonlinear differential equations through linear approximations is often criticized for its resulting multiple (dis)equilibria (e.g., Nijkamp and Reggiani 1998) and for failure to account for discontinuities. Several studies have generalized models of new product growth to capture the effect of sales volatility by adding a stochastic term to the sales dynamics equation (see, e.g., Boswijk and Franses 2005).

To track the mechanism through which new products penetrate the marketplace, several attempts have been made to divide the total market into several pieces and model the interactions between them. For example, there is a growing consensus on the fundamental role that the structure of social networks play in how information reaches consumers, channel members, and suppliers. This issue is particularly important in the marketing of new products and the creation of marketing collaborations (Iacobucci 1996, Achrol and Kotler 1999, Rosen 2000). Recent attempts have been made to directly tie social network properties to success in marketing activities such as pricing or promotion strategies (Mayzlin 2002, Shi 2003). Yet much of the empirical marketing research in this area has focused on relatively small networks, for example, intra- or interorganizational networks (see Houston et al. 2004 for a review), tie strength (Brown and Reingen 1987, Rindfleisch and Moorman 2001), or social capital (Ronchetto et al. 1989).

A possible reason for this might be that in most of the literature, for practical reasons, the market is assumed to be homogeneous given that networks and other more complex structures make modeling and estimation procedures much more complex. For instance, uncovering market structures for new product introduction in international markets (i.e., "international diffusion") is a research branch where issues such as which market should be penetrated first (i.e., countries with a higher connectivity level, etc.) are of interest. Putsis et al. (1997) addressed the question of how adoption in one country affects adoption in others by uncovering the importance of a mixing (interaction between countries) pattern that is grounded in communication within and across countries. While this is an important direction to pursue, the size of the network used in that approach, the empirical treatment (i.e., several dozens of nodes), and the resolution (not at the consumer level; i.e., nodes are countries and not consumers) are rather limiting.

Similar treatment of diffusion modeling and networks can be found in the business-to-business area. For example, Jones and Ritz (1991) suggested a stage-based diffusion, with the organization first and then the individuals within the organization (for a different model, see Kim and Srivastava 1998). Here again, the attempt is to add structure to the diffusion model, yet the scale size is at a level of a few units, thus preventing us from gaining significant insight into consumer networks or those individuals who maintain a significant, large number of social ties. In sum, lack of large-scale observations and nonconsumer-level treatments are drawbacks of attempts in this general direction of modeling.

Another important basis for the argument for consumer segmentation in the diffusion literature is the classification of consumers into various types of adopters. Recently, Van den Bulte and Joshi (2007) introduced a model of growth using two adopter segments: influentials and imitators. The former affect the latter segment, yet not vice versa. This two-segment structure with asymmetric influence is consistent with several theories in sociology and diffusion research.

Other attempts to understand how networks affect growth processes exist in the literature and are characterized by an integrated, direct approach wherein both the aggregate growth process and the individual level of the network are analyzed. Reingen and Kerman (1986), for example, demonstrate that the role of subgroups in referral to network flow is typified by small social groups (see also Goldenberg et al. 2009). Brown and Reingen (1987) looked at word-of-mouth (WOM) referral behavior in a natural environment. An interpersonal network was examined, and the various roles and effects related to tie strength were found. These findings, however, are limited in the number of possible segments and do not focus on individual customers. As such, the findings are limited in their ability to directly refer to the existence and influence of consumer hubs. Nevertheless, the rationale of these studies indicates that with richer data, focus on the consumer as a unit analysis in the model might lead to better understanding of such a process.

Several studies have directly addressed the diffusion process from an individual-level point of view. Horsky (1990) introduced an individual-level model describing a durable purchase decision made by a utility-maximizing household, based on the household's income, the benefits derived from the product, and the product's price. An aggregate demand equation was then derived and shown to exhibit a sales curve in a product life-cycle pattern. Chatterjee and Eliashberg (1990) also presented a micromodeling approach that explicitly considers the determinants of adoption at the individual level. Consumers' preference structure was formulated by utility function in a von Neumann-Morgenstern framework to encapsulate the potential adopters' risk aversion. Eliashberg et al. (2000) designed a decision-support system for prerelease market evaluation of motion pictures based on a Markov process framework to estimate behavioral factors such as WOM and movie-specific parameters. Reformulating the Bass model as a pure birth process, Niu (2002) showed that the birth probability of a new product adopter converges to the solution of the Bass differential equation when the market consists of identical consumers, and its potential goes to infinity. These studies clearly highlight the potential for modeling the diffusion process at the individual level. However, none made use of granular data, nor did they take into account the effects of networks on the diffusion patterns. Therefore, neither the movements nor how predictions can be improved by using this characteristic have been addressed.

In this paper, we zoom in to a granular resolution of data to better capture the individual-level unit of analysis and develop a methodological framework. Richer data and focus on the consumer as a unit of analysis can alleviate some of the limitations of more traditional models, as demonstrated in this paper.

2. The Methodological Framework

2.1. An Individual Analysis Framework

One of the objectives of this study is to analyze the emergence of movements in firms' sales. Our analysis is based on the idea of *stochastic cellular automata* introduced by Goldenberg et al. (2001b), wherein the probabilistic behavior of the individual consumer is embodied by an agent-based model to simulate the dynamics of new product growth. Along these lines and unlike aggregate approaches, our unit of analysis is the individual consumer. Let s_i be a binary variable that represents the state of adoption of a potential customer i. That is, $s_i(t)$ takes the value 1 if an individual i had adopted the innovation before time t and 0 otherwise. We define the vector $\vec{S}(t) = [s_1(t), s_2(t), \ldots, s_M(t)]$ as the market conditions vector at time t, where M is the market potential. A customer status change from a potential adopter to an actual adopter is based on the transition from the state of not-adopting yet to the state of adopting (a transition from $s_i = 0$ to $s_i = 1$).

We postulate that the transition process between *potential* adopter to adopter state is stochastic in nature and occurs with time-dependent probabilities $\eta_i(t)\Delta t$ ($i = 1, 2, \ldots, M$), where $\eta_i(t)\Delta t$ is the probability that individual i will adopt the innovation within the time interval between the time indexes t and $t + \Delta t$. These probabilities are determined by market conditions that affect an individual's decision whether or not to adopt the innovation. The literature on new product growth identifies two main forces that govern the penetration rate in such a case. These are marketing efforts (also termed an external force) and WOM (also termed an internal force and including interactions between consumers such as referrals, imitations, etc.). The external force that affects the potential adopter i can be represented by the probability $p_i(t)\Delta t$ that this individual will adopt the innovation within a time interval of Δt after the time t because of the influence of advertising and mass media. The internal force that affects the potential adopter can be represented by a set of probabilities $q_{ij}(t)\Delta t$, where $q_{ij}(t)\Delta t$ is the probability that a potential customer i will adopt the innovation within a time interval Δt after time t as a result of WOM communication provided by customer j. It is a common assumption that these forces are orthogonal. In this case, the transition probability for an individual to adopt the innovation within a time interval Δt after the time t is given by

$$\begin{aligned}\eta_i(t)\Delta t &= 1 - [1 - p_i(t)\Delta t] \cdot \prod_{j \neq i} [1 - q_{ij}(t)\Delta t] \\ &= \left[p_i(t) + \sum_{j \neq i} q_{ij}(t)\right]\Delta t + O((\Delta t)^2). \qquad (1)\end{aligned}$$

This equation is a generalization of adoption and growth mechanisms, and it is consistent with previous studies (e.g., Goldenberg et al. 2001a, Garber et al. 2004). These probabilities are conditional in nature and in general depend on the relevant history of the entire market dynamics (i.e., time dependent). More formally, the transition probabilities can be defined as

$$\eta_i \Delta t \equiv \eta_i(t \mid \Omega_t)\Delta t, \qquad (2)$$

where $\Omega_t = (\vec{S}(t), \vec{S}(t - \Delta t), \ldots, \vec{S}(0))$ is the history of the past market condition vectors. For example, under the assumption that the transition probabilities depend only on the current market state vector $\vec{S}(t)$ via WOM communications and where only an adopter can influence another potential one, Equation (1) can take the form

$$\begin{aligned}\eta_i(t \mid \Omega_t)\Delta t &= \eta_i(t \mid \vec{S}(t))\Delta t \\ &= \left(p_i(t) + \sum_{j \neq i} w_{ij}(t) s_j(t)\right)\Delta t, \qquad (3)\end{aligned}$$

where $s_j(t)$ is the jth component of the market condition vector $\vec{S}(t)$ and $q_{ij}(t) = w_{ij}(t)s_j(t)$, so that $w_{ij}(t)$ is the probability per unit of time that potential adopter i will adopt the innovation because of WOM influence produced by customer j, who had already adopted the new product.

The probability of a potential adopter deciding to adopt the innovation within time interval Δt after a certain time period, t, can be also expressed in terms of transition probabilities. Given Ω_t, the history of the past market condition vectors in a given time, t, the probability that an individual i will be an adopter at the time $t+\Delta t$ is

$$\Pr(s_i(t+\Delta t)=1 \mid \Omega_t)=\begin{cases}1 & \text{if } s_i(t)=1,\\ \eta_i(t\mid\Omega_t)\Delta t & \text{if } s_i(t)=0.\end{cases} \tag{4}$$

Because our interest is exploring the dynamic process of penetration of the new product into the marketplace, we define a transition index of purchasing for a potential customer i as

$$\Delta s_i(t)=s_i(t+\Delta t)-s_i(t) \tag{5}$$

to obtain the purchasing conditional probability,

$$\Pr(\Delta s_i(t)=1\mid\Omega_t)=(1-s_i(t))\eta_i(t\mid\Omega_t)\Delta t, \tag{6}$$

where $\Pr(\Delta s_i(t)=1\mid\Omega_t)$ is the probability that a potential customer will adopt the innovation within the time interval between t and $t+\Delta t$, given the relevant history of the market dynamics at the time t. Since Δs_i is a binary variable, $\Pr(\Delta s_i=0)=1-\Pr(\Delta s_i=1)$ is also true. Alternatively, we can also rewrite Equation (6) to obtain the transition index:

$$\Delta s_i(t) = \begin{cases}1 & \text{with probability } (1-s_i(t))\eta_i(t\mid\Omega_t)\Delta t,\\ 0 & \text{otherwise.}\end{cases} \tag{7}$$

The transition indices of purchasing $\Delta s_i(t)$ are stochastic variables in nature and can be considered independent, as if the time interval Δt is set to a very small value in which the probability of potential adopters to communicate with adopters is limited. The resulting WOM effect to induce purchasing during the short time period Δt becomes negligible.

2.2. An Aggregate-Level Point of View

The first step in bridging between the micro- and the macropoints of view is to define the noncumulative penetration of the new product. This will be the number of individuals who have adopted the innovation within a given short time interval Δt after a certain time t and can be expressed by the sum of all individual transition indices of purchasing the product such that

$$\Delta m(t)=\sum_{i=1}^{M}\Delta s_i(t). \tag{8}$$

The noncumulative penetration is a stochastic variable and consists of the sum of independent binomially distributed variables. Given the history of the past market condition vectors Ω_t at time t, the noncumulative penetration mean and variance can be derived. Under the assumption that simultaneous purchases are uncorrelated within a short time interval Δt, the mean is given by

$$\Delta\overline{m}(t\mid\Omega_t)\equiv E(\Delta m(t)\mid\Omega_t)=\sum_{i=1}^{M}(1-s_i(t))\eta(t\mid\Omega_t)\Delta t, \tag{9}$$

and the variance is

$$\begin{aligned}\sigma^2(t\mid\Omega_t)&=E([\Delta m(t)-\Delta\overline{m}(t)]^2\mid\Omega_t)\\ &=\Delta\overline{m}(t\mid\Omega_t)-\sum_{i=1}^{M}(1-s_t(t))[\eta_i(t\mid\Omega_t)\Delta t]^2\\ &=\Delta\overline{m}(t\mid\Omega_t)+O\left(\frac{1}{M}[\Delta\overline{m}(t\mid\Omega_t)]^2\right),\end{aligned} \tag{10}$$

where $E(.\mid\Omega_t)$ denotes a conditional expected value given the history Ω_t of the past market condition vectors at the time t.

The next step is formulating the aggregate-level dynamics of a new product growth. For that purpose we construct a dynamic equation of the new product penetration that is governed by the sum of all market forces (externals and internals), which affect each individual in the marketplace. These market forces determine the probability of adoption of each potential consumer and are defined by the sum of the conditional probabilities of adoption per unit of time over all potential adopters in the market. This sum is equivalent to the average penetration rate, so the net market force at time t can be written as

$$F(t,\Omega_t)\equiv\sum_{i=1}^{M}(1-s_i(t))\eta_i(t\mid\Omega_t)=\frac{\Delta\overline{m}(t\mid\Omega_t)}{\Delta t}, \tag{11}$$

and the dynamics of the noncumulative penetration can be described by

$$\Delta m(t)=F(t,\Omega_t)\Delta t+\varepsilon_t, \tag{12}$$

where ε_t is uncorrelated noise with mean zero and variance determined by the net market force. That is, the following conditions are satisfied: $E(\varepsilon_t\mid\Omega_t)=0$, $E(\varepsilon_t^2\mid\Omega_t)=F(t,\Omega_t)\Delta t$, and $E(\varepsilon_t\varepsilon_{t'}\mid\Omega_t)=\varepsilon_{t'}E(\varepsilon_t\mid\Omega_t)=0$ for $t>t'$. Note that given $\Omega_{t'}$, a past noise term $\varepsilon_{t'}$ (where $t'<t$) is already determined and has a fixed value.

The actual net market force F (Equation (11)) is defined as a function of Ω_t, the history of the past market condition vectors. These vectors should contain individual-level information and, as a result, impose a significant restriction on the applicability of such a modeling approach; as such, data are seldom available. Nevertheless, we can use Equation (12) (noncumulative sales) and rewrite it to come up with the following expression with partial information:

$$\Delta m(t)=\hat{F}(t,\hat{\Omega}_t)\Delta t+u(t), \tag{13}$$

where $\hat{F}(t,\hat{\Omega}_t)$ is the model of the net market force which estimates the actual net market force F on the basis of the partial information $\widehat{\Omega}_t$. In contrast, the term $u(t)$ denotes the actual noise, or stochasticity, of the process and is given by

$$u(t) = (F(t, \Omega_t) - \hat{F}(t, \widehat{\Omega}_t))\Delta t + \varepsilon_t, \tag{14}$$

where the first and second moments of the penetration rates' actual noise satisfy the following relations:

$$E(u(t) \mid \Omega_t) = (F(t, \Omega_t) - \hat{F}(t, \widehat{\Omega}_t))\Delta t, \tag{15}$$

$$E([u(t)]^2 \mid \Omega_t) = F(t, \Omega_t)\Delta t + [(F(t, \Omega_t) - \hat{F}(t, \widehat{\Omega}_t))\Delta t]^2 \tag{16}$$

and

$$E(u(t)u(t') \mid \Omega_t) = u(t')E(u(t) \mid \Omega_t). \tag{17}$$

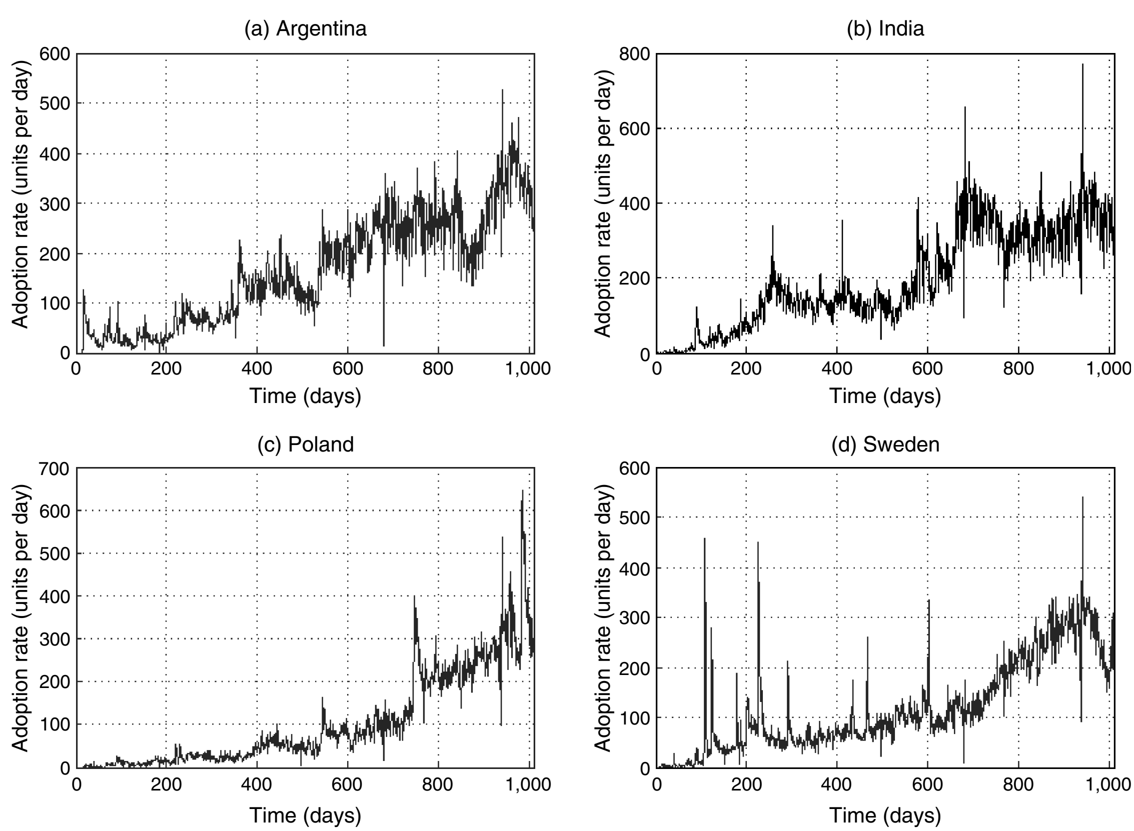
Note that the past noise term $u(t')$, where $t' < t$ is already determined, is fixed at the present time t. The actual noise contains all the relevant microlevel information that has not been (or cannot be) modeled. Since, in general, $E(u(t) \mid \Omega_t) \neq 0$ and $E(u(t)u(t') \mid \Omega_t) \neq 0$ for time indexes t and t', where $t > t'$, the actual noise in the noncumulative penetration usually becomes biased and correlated. As a result, strong coupling effects can be observed in the sales curve. These effects can be interpreted as large fluctuations or trends in sales—our argument for the internal emergence of penetration tends. Specifically, the noise can be a result of either or both systematic (i.e., a change in the trend of the penetration function) and nonsystematic (i.e., fluctuations resulting from various market or individual conditions). That is, modeling the actual net market force F exhibits a trade-off. The simpler and hence the less microinformative the model of the net market force $\hat{F}$, the more significant the impact of the noise u in producing large fluctuations and changing trends in the penetration data.

In sum, through aggregation of a microscopic approach (also known as agent-based modeling), we get a collective behavior; i.e., we developed a general framework wherein the penetration can be modeled. In that context, we demonstrate in the appendix the use of the proposed framework as a general modeling tool for the process of new product adoption, while deriving the fundamental Bass model as a special case.

3. Self-Emergence of Penetration Movements

Various types of penetration patterns can be observed when analyzing the noncumulative penetration of innovative products. To illustrate, consider Figure 2, wherein data of four cases of penetration of the e-mail

Figure 2 **The Actual Daily Penetration of an E-mail Software Product in Four Different Markets: (a) Argentina, (b) India, (c) Poland, and (d) Sweden**



software tool in various countries are shown. The noncumulative penetrations are presented in a daily time frame for Argentina, India, Poland, and Sweden during the 1,010-day period from September 1, 2000 to June 7, 2003. A primary tendency of increasing adoption rates is evident in all the cases. Secondary movements, however, can also be observed. For example, in Argentina (Figure 2(a)), chapel-like patterns of about 100–200 days' width each climb over the increasing primary movement of adoption rates. In the case of India (Figure 2(b)), changing movements (trends) in the penetration curve form shapes that look like pointed minarets and can be observed in intervals of several hundred days. An abrupt jump can be identified in the Polish penetration curve (Figure 2(c)), and the moderate monotonic increase in penetration in the Swedish market (Figure 2(d)) is almost suddenly replaced by steep growth that reaches its peak before experiencing a steep decline. Regularities in the patterns of penetration can also be visually identified in shorter time periods on the scale of dozens of days.

3.1. Study 1: Analysis of the Empirical Relationship Between the Volatility and Mean of the Adoption Rates

To obtain the mean and standard deviation of adoption rates, we retrieve empirical measures and statistical estimators for these variables in a given averaging time window.

Let $\{\Delta m(t)\}$ be the series of daily adoption rates of a new product such that $\Delta m(t)$ is the number of purchases that occurred during a time period of one day after time t. Then, for any given time t, we can calculate the following statistical estimates:

$$\Delta\hat{\bar{m}}(t) = \frac{1}{\Lambda}\sum_{\lambda=0}^{\Lambda-1}\Delta m(t+\lambda\Delta t) \tag{18}$$

and

$$\hat{\sigma}(t) = \left[\frac{1}{\Lambda-1}\sum_{\lambda=0}^{\Lambda-1}(\Delta m(t+\lambda\Delta t) - \Delta\hat{\bar{m}}(t))^2\right]^{1/2}, \tag{19}$$

where $\Delta\hat{\bar{m}}(t)$ and $\hat{\sigma}(t)$ are the estimators for the mean and standard deviation of the noncumulative penetration data, respectively. In the data we analyzed, the time interval is $\Delta t = 1_{\text{day}}$. The averaging window $\Lambda\Delta t$ determines the time scale, such that below this window's size any observed oscillation of the penetration data can be considered fluctuation.

We assume that during a short time interval (one day) simultaneous purchases are independent; thus Equation (10) can be used to describe the relationship between the adoption rates' mean and standard deviation. We therefore replace the expected values by their statistical estimates and obtain the following approximation:

$$\hat{\sigma}(t) \approx (\Delta\hat{\bar{m}}(t))^{1/2} \tag{20}$$

In our empirical validation, for each data set we use an ordinary logarithmic regression of the form

$$\hat{\sigma}(t) = A(\Delta\hat{\bar{m}}(t))^{\alpha}, \tag{21}$$

where the regression coefficients are predicted by Equation (20) to be $A = 1$ and $\alpha = \frac{1}{2}$.

The goodness-of-fit results of this estimation process are given in Table 1, where the averaging window parameter Λ was taken to be 10, such that $\Lambda\Delta t = 10_{\text{days}}$. In general, it can be seen that our logarithmic regressions nicely fit the data. The R^2 values vary from 0.52 to 0.83, with an average of 0.7. The regression coefficients usually lie around their predicted values. Yet it is reasonable that Equation (20) underestimates the penetration volatility. The power α is approximately 0.6 instead of the expected 0.5, and the factor A tends to be larger than 1. (The coefficient errors were calculated for a 10% significance level as if the regression residuals are normally distributed.)

Table 1 **The Noncumulative Penetration Volatility vs. the Noncumulative Mean of Penetration of the Tested Software Tool—Fitting Results**

Country	A	α	R^2
Argentina	1.12 [0.76, 1.51]	0.63 [0.55, 0.71]	0.667
India	1.11 [0.86, 1.42]	0.65 [0.60, 0.71]	0.833
Poland	0.95 [0.75, 1.19]	0.63 [0.57, 0.69]	0.794
Sweden	1.50 [1.00, 2.26]	0.55 [0.46, 0.64]	0.517

There are two main reasons for the effective expansion of the penetration volatility. First, we assume that the net market force is constant within the averaging window time period (i.e., adoption rates are generated by the same distribution). This assumption, however, does not consider possible short-term movements during the averaging window time period. Such movements might occur even within such short time intervals because of the exact timing of product purchases (e.g., weekend versus midweek sales), sudden changes in the collective market behavior because of external effects (e.g., a reaction to breaking news or rumors in the media), or a firm's actions (e.g., promotions). Second, violation of the simultaneous purchase independence assumption that forms the basis for Equations (10) and (20) can be another source for expansion of sales volatility. For example, a market that consists of a social network of relatively small groups (i.e., segments), wherein the social relations are very strong, can produce highly correlated purchases of the new product. In the next study, we demonstrate this phenomenon by using an agent-based model.

3.2. Study 2: Demonstration of the Process of Movements Creation via the Stochasticity of the New Product's Growth

This study will is to show that the emergence of changes in sales trends can be a fundamental feature of a product's penetration into the marketplace and does not necessarily have to be a result of external influence. Such movements can be found even in homogeneous markets with fixed market forces. We studied the apparent difference between the deterministic-continuous and the stochastic-discrete analyses of such a process by using analytical calculations as well as numerical simulations.

For illustration purposes, we consider the simple case of a market comprised of identical consumers. We also consider the market to be constructed by several segregated submarkets of consumers having social links.[2] We further assume that the social interactions among individuals within the same submarket are much stronger than those among individuals coming from different submarkets.

The market consists of M consumers (i.e., M is the market potential), where each consumer can purchase the innovation just once. The market is segregated into N submarkets, where M_v denotes the number of individuals within the vth submarket. To simplify our analysis, we further assumed that all submarkets are of the same size (i.e., for any submarket, v, $M_v = (1/N)M$).

External force. For any individual i there exists a probability $p_i(t)\Delta t$ of being influenced by external forces (e.g., advertising) to adopt the innovation within a short time interval Δt after time t. ($p_i(t)$ denotes a probability per unit of time.) We assume that these probabilities are identical (equal) for all consumers and do not vary with time (i.e., $p_i(t)\Delta t \equiv p\Delta t$ for any individual i).

Internal force. Among any individual pair i and j, a probability $q_{ij}(t)\Delta t$ exists that individual j who has already adopted the product will interact with and influence individual i to adopt the innovation within a short time interval Δt after time t as a result of WOM communication ($q_{ij}(t)$ denotes a probability per unit of time). We assume that social links among individuals are identical and fixed. We consider two types of social links: intrasubmarket links (i.e., representing the strong social relations between two individuals in the same group, depicted by a probability constant $q_s\Delta t$) and intersubmarket links (i.e., representing the weak social relations among individuals belonging to different social groups and expressed by a probability constant $q_w\Delta t$, which generally satisfies: $q_w\Delta t \ll q_s\Delta t$). That is to say, $q_{ij}(t)\Delta t \neq 0$ only in a case where individual j has purchased the innovation until time t. In that case, $q_{ij}(t)\Delta t = q_s\Delta t$, when both individuals i and j are part of the same social group and, hence, are connected via intrasubmarket link and $q_{ij}(t)\Delta t = q_w\Delta t$ otherwise (when they belong to differing social groups and are tied by an intersubmarket link).

We can now derive the individual's transition probability from a nonadopting state to an adopting state. Let i be a potential adopter who belongs to the vth submarket. Then, the probability that an individual will adopt the innovation within the time interval that lies between t and $t + \Delta t$ is given by Equation (1) as follows:

$$\begin{aligned}\eta_i(t)\Delta t &= 1 - (1 - p_1(t)\Delta t) \cdot \prod_{j\neq i}(1 - q_{ij}(t)\Delta t)\\ &= 1 - (1 - p\Delta t)(1 - q_s\Delta t)^{m_\nu(t)}(1 - q_w\Delta t)^{m(t)-m_v(t)}\\ &\equiv \eta^{(\nu)}(t)\Delta t, \end{aligned} \tag{22}$$

where $m_v(t)$ is the number of individuals in the vth submarket who had already adopted the innovation by time t and, hence, denotes the cumulative amount of sales in the vth submarket through that time t. The value of $m(t)$ denotes the entire market's cumulative sales, where $m(t) = \sum_{v=1}^{N} m_v(t)$.

Next, we calculate the net market force applied on the vth submarket at time t. This net market force is, by definition, given by Equation (11) and is repeated here as

$$F_v(t) = \sum_{i\in SM_\nu} (1 - s_i(t))\eta_i(t). \tag{23}$$

SM_v denotes the class of individuals belonging to the vth submarket, and $s_i(t)$ is the ith individual's state of adoption. Because all the transition probabilities within a certain submarket v are equal, substituting (22) in (23) yields

$$\begin{aligned}F_v(t) &= \left(\frac{M}{N} - m_v(t)\right)\frac{\eta^{(v)}(t)\Delta t}{\Delta t}\\ &= \left(\frac{M}{N} - m_v(t)\right)\\ &\quad\cdot \frac{1 - (1 - p\Delta t)(1 - q_s\Delta t)^{m_\nu(t)}(1 - q_w\Delta t)^{m(t)-m_v(t)}}{\Delta t}, \end{aligned} \tag{24}$$

[2] Our definition of *submarket* is similar to a network, albeit more general. We do not use the term "network," as networks are generally viewed as clusters with strong ties inside. Our model, however, does not follow exactly the definitions of strong and weak ties, because they are not required for this purpose. A submarket is a group that posteriori can be viewed as an "activated network." The overall effect of both strong and weak ties is taken into account as a complete adoption process inside this group. The summation of all groups comprises the overall adoption process. From this perspective, some groups can be connected geographically (as shown already in the literature), whereas others can be connected through other channels of information.

where N is the number of submarkets and M is the total market potential. We can now use the penetration dynamics equation (Equation (12)) to obtain the noncumulative penetration dynamics in the vth submarket

$$\Delta m_v(t) = \Delta \overline{m}_v(t) + \varepsilon_v(t). \tag{25}$$

Here, $\Delta \overline{m}_v(t)$ is the mean value of the adoption rate at time t within the vth submarket and is given by

$$\Delta \overline{m}_v(t) = F_v(t)\Delta t. \tag{26}$$

The term $\varepsilon_v(t)$ denotes the stochasticity of the process. It represents an uncorrelated noise with a mean of zero and standard deviation approximately given by the square root of the noncumulative sales average (i.e., $E([\varepsilon_v(t)]^2) \approx \Delta \overline{m}_v(t)$). Specifically, each one of the noncumulative penetration variables $\Delta m_v (v = 1, 2, \ldots, N)$ is composed of Δm_v positive outcomes that have been taken from a sample of $M/N - m_v$ identical binary random variables, where each positive outcome occurs with a probability of $\eta^{(v)}\Delta t$. Thus according to our model, the noncumulative penetration variables Δm_v are binomially distributed and hence the noise terms are $\varepsilon_v (v = 1, 2, \ldots, N)$ drawn from a shifted binomial distribution.

To obtain further insight into the dynamics of new product growth, we first look at the deterministic part of the dynamics of this process. Here, we omit the noise terms and set the problem to the continuous limit (i.e., taking the short sampling interval Δt to zero). In this case, for any submarket v, Equation (25) becomes an ordinary differential equation of the form

$$\frac{dm_v(t)}{dt} = F_v(t), \tag{27}$$

where

$$F_v(t) \xrightarrow[\Delta t \to 0]{} \left(\frac{M}{N} - m_v(t)\right)(p + (q_s - q_w)m_v(t) + q_w m(t)). \tag{28}$$

If the initial time $t = 0$ is the time of the new product launch, we can solve the following N ordinary differential equations system:

$$\frac{dm_v(t)}{dt} = \left(\frac{M}{N} - m_v(t)\right)(p + (q_s - q_w)m_v(t) + q_w m(t)), \tag{29}$$

where $m(t) = \sum_{v=1}^{N} m_v(t)$ and the initial conditions are $m_v(t = 0) = 0$ for $v = 1, 2, \ldots, N$. As the dynamic equations are identical for all submarkets as well as the initial conditions, we can deduce that the solutions are identical for all the submarkets. As a result,

$$m_v(t) = \frac{1}{N} m(t) \tag{30}$$

for any submarket v. It follows that if we substitute relations (30) in Equation (29), we find that under the deterministic-continuous limit of our model, the dynamics of penetration are given by the Bass model

$$\frac{dm(t)}{dt} = (M - m(t))\left(p + \frac{Q}{M} m(t)\right), \tag{31}$$

where

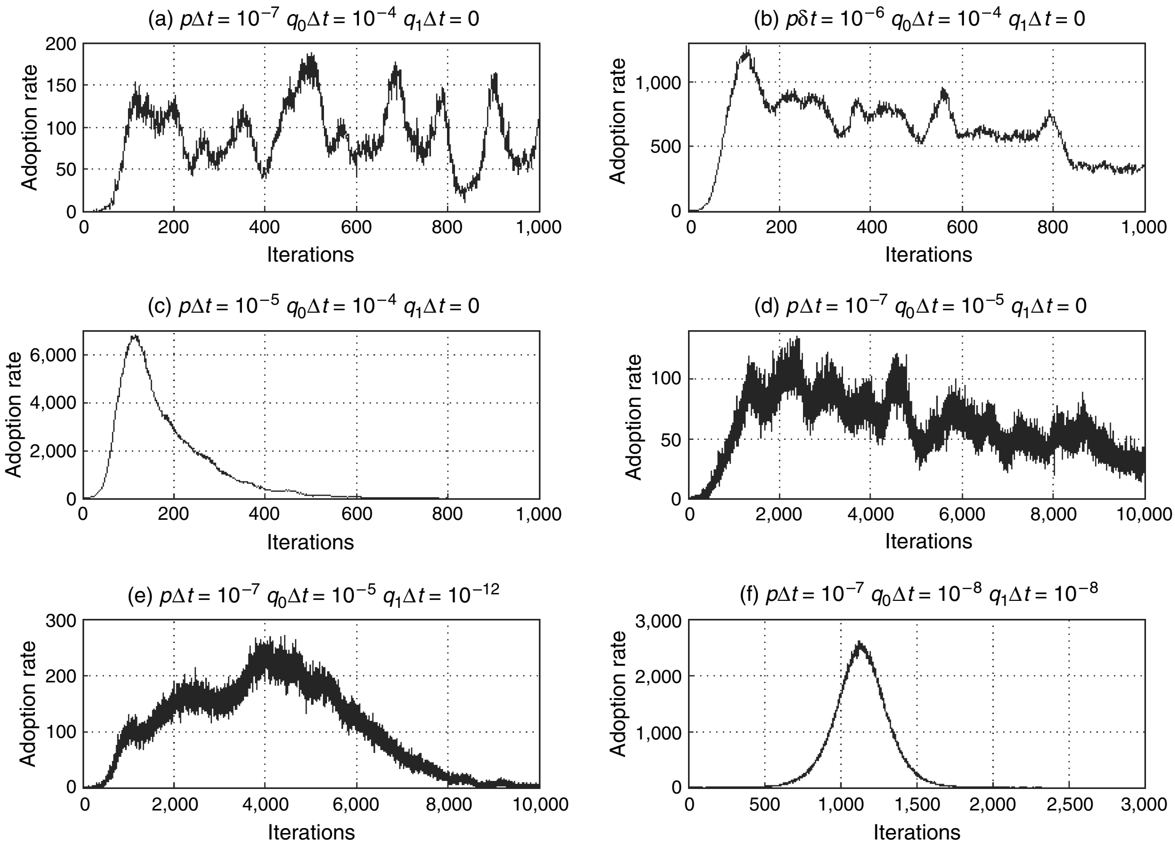
$$Q = M\left(\frac{1}{N} q_s + \left(1 - \frac{1}{N}\right) q_w\right), \tag{32}$$

and $P = p$. Thus, taking only deterministic considerations in our model into account, we expect the evolution of penetration to exhibit the traditional bell-shaped curve with a single peak at time t^* given by the Bass equation solution. Furthermore, Equation (32) predicts a similar effect on adoption rates for interactions of individuals of both within a submarket and between different submarkets. According to the deterministic analysis, the main difference between these two types of interaction is merely quantitative. That is, alternating the intensity of the intersubmarket link q_w is equivalent to $N - 1$ times stronger modification of the intrasubmarket link q_s.

We now examine the influence of the inherent stochastic process on the evolution of the sales process. To this end, we used an agent-based simulation (see, e.g., Goldenberg et al. 2002) and define M binary variables (agents) to represent the state of consumption of each individual in the simulated market. We also classify those variables into N equal-sized classes to designate N submarkets of equal size. The state values of consumption are updated via iterated probabilistic dynamics defined by the individual-level probabilities of adoption presented in Equation (22). In each iteration, we realize the temporal evolution of the adoption rates in the market during a short time period of length Δt. The external influence probability $p\Delta t$, as well as the internal influence probabilities within and among submarkets $q_s\Delta t$ and $q_w\Delta t$, respectively, are fixed parameters of the simulation.

Figure 3 illustrates the influence of stochasticity on the process of new product growth, where the resulting penetration curves of six numerical realizations are presented. Six choices of the external influence parameter $p\Delta t$, the intrasubmarket, and the intersubmarket links $q_s\Delta t$ and $q_w\Delta t$ were used. Each realization emulates the temporal evolution of penetration in short sampling intervals (say, days) in a market consisting of $M = 10^6$ consumers, classified into $N = 1{,}000$ equal-sized submarkets. Unlike the prediction of the deterministic-continuous analysis of this process, some of the presented cases do not exhibit the traditional bell-shaped curve. There are cases wherein the new product penetration dynamics create an oscillating pattern of penetration (see, e.g., Figure 3(a)).

Figure 3 Demonstration of the Process of Penetration Movements' Creation: Curves of Six Numerical Realizations of an Agent-Based Simulation of the Multisubmarket Configuration



In other cases, changes in penetration trends are carried as secondary movements on a primary decreasing line (see, e.g., Figures 3(b) and 3(d)). The existence of other than bell-shaped patterns in a homogeneous and stationary market is the sole result of the stochastic nature of the purchasing process.

The results of the stochastic effects on the process of new product growth are depicted by the temporal evolution of penetration in various submarkets that are no longer identical. Recall that the dynamics of penetration in a certain submarket v as presented in Equation (25) include a provision for the stochasticity of the process by including the noise term. Naturally, the noise terms ε_v of various submarkets $v = 1, 2, \ldots, N$ usually differ from one another. Thus, the values of the noncumulative penetration Δm_v and therefore those of the cumulative penetration Δm_v are not identical in all submarkets.

Similarly, the net market forces in various submarkets at a given time t also differ, because they directly depend on the cumulative penetration values (see Equation (24)). Thus, the diversity of various growth forms is extended even further. In contrast, when assuming a deterministic motion, all submarkets evolve through the same dynamic process and, as a result, exhibit an identical curve. This curve is produced by typical Bass dynamics, as indicated by Equation (31).

The stochasticity of the process impacts the dynamics of new product growth in two primary ways. First, it breaks the simultaneity of the temporal evolution of adoption rates in various submarkets: various smaller submarkets respond according to differing launch times. In general, a specific submarket can exhaust its sales potential before initial adoption of the product has started in another submarket. As a result, the dynamics of adoption rates in the entire market may exhibit changes in penetration movements. Furthermore, stochastic effects may produce movements even when the processes evolve simultaneously in differing submarkets. The dynamics of new product growth are multiplicative. Specifically, the larger the number of customers who have already adopted the innovation within a certain submarket, the larger (on average) the number of customers who adopt the innovation at present time t in the penetration stage of the new product. (This property is straightforwardly derived from the structure of the net market force applied on a certain submarket v at a given time t that determines the mean value of the noncumulative penetration as shown by Equations (24) and (26).)

Consequently, initial differences in adoption rates in differing submarkets that were originally triggered by stochastic effects increase by the multiplicative dynamics of the new product's growth. Penetration within submarkets wherein the initial number of individuals who adopted the innovation was high by chance alone, will reach their peak faster than will sales within submarkets with a low initial number of adopters. Such cases will lead to market-level penetration patterns that may hold several maximum and minimum values.

Let us now identify the effects of each of the process parameters. Figures 3(a)–3(c) illustrate the effect of external parameter $p\Delta t$ on the dynamics of market sales, and present the penetration curves of the isolated submarket's case. Specifically, there are no interactions among individuals from differing submarkets, so the intersubmarket links are nullified (i.e., $q_w\Delta t = 0$). The intrasubmarket links are equal and are given by $q_s\Delta t = 10^{-4}$. The range of the external parameter $p\Delta t$ varies from 10^{-7} to 10^{-5}. As our iteration interval emulates daily sampling, those values are analogous to annual rates of the P-parameter in the range of 0.000036 to 0.0036. When the value of $p\Delta t$ is small, the adoption rate fluctuates upward and downward, displaying an oscillating pattern (see Figure 3(a), where $p\Delta t = 10^{-4}$). Gradually, when the value of $p\Delta t$ increases, a primary declining movement in penetration is revealed (see Figure 3(b), where $p\Delta t = 10^{-6}$) that becomes the traditional bell-shaped curve (as shown in Figure 3(c), where $p\Delta t = 10^{-5}$).

To obtain more insight into the resulting patterns, we recall that when external parameter is $p\Delta t$ large, a high probability exists of the new product simultaneously penetrating into numerous submarkets. Once $p\Delta t$ is very small, the probability of a given submarket being activated at a given time is very small, and only a few submarkets will be active at the same time. Under such conditions, a number of peaks can be found because penetrations in differing submarkets are effectively evolved serially. In the case of large $p\Delta t$, the growth process tends to evolve simultaneously in more than several submarkets, and we begin observing the traditional bell-shaped curve. This is the result of the relationship between the stochastic and deterministic terms. As the stochastic term becomes smaller relative to the deterministic component, the bell-shaped sales curve emerges.

Figures 3(d) and 3(a) together depict how the ties among individuals within the same submarket affect the penetration dynamics throughout the market. Here, the intrasubmarket link is weaker (i.e., $q_s\Delta t = 10^{-5}$ instead of $q_s\Delta t = 10^{-4}$), as in Figure 3(d). It can be seen that the duration of the change in sales trends develops into longer patterns (from an order of 100 iterations when $q_s\Delta t = 10^{-4}$, to about 1,000 iterations when $q_s\Delta t = 10^{-5}$). A primary movement that indicates a gradual decline in sales becomes apparent, and the variance of the fluctuations in the process becomes large. This is the result of a decrease in the intrasubmarket link that slows the evolution of penetration into each submarket.

The lifespan of each submarket, therefore, is extended, as are the sales trends. Furthermore, the lower the number of intrasubmarket links, the higher the number of "active" submarkets at any given time. Thus, the effect of breaking the penetration simultaneity discussed earlier is reduced, and a decline in the primary line of sales appears. The increase in the variance of the fluctuations is a result of the combined effect of a decrease in each market's mean value of sales because of the decreasing effect of the internal force (i.e., word-of-mouth communication) and concurrent processes in many submarkets. The total sum that defines the noncumulative penetration of the entire market, therefore, is composed of a higher number of noisy terms in relation to the number of "active" submarkets, where the standard deviation of each one approaches the mean.

Of particular interest is the comparison between the apparent penetration curves in Figures 3(d) and 3(e). Instead of considering the assortment of isolated submarkets, we used very weak social ties among individuals coming from differing submarkets. The intensity of $q_s\Delta t = 10^{-12}$ the intersubmarket link is defined for the Case 3E to be rather than zero, where all the other parameters remain identical. Surprisingly, the resulting impact on the pattern of penetration that follows this tiny change is significant. The gradual decline of sales when submarkets are isolated (Figure 3(d)) is replaced by a well-defined primary movement with a global peak (Figure 3(e)).

This phenomenon underscores the effect of stochasticity on this process. If we take only deterministic considerations into account, both cases should exhibit the same curve. As mentioned earlier, both the intrasubmarket links q_s and the intersubmarket links q_w affect the dynamics of penetration via the WOM coefficient Q given in Equation (32). Substituting the appropriate numerical values of the parameters for both cases (i.e., $M = 10^6$, $N = 10^3$, and $q_s\Delta t = 10^{-5}$ in the two cases; $q_w\Delta t = 0$ in Case 3D; and $q_w\Delta t = 10^{-12}$ in Case 3E) yields the same WOM coefficient $Q\Delta t = 10^{-2}$. Namely, the deterministic analysis cannot distinguish between intrasubmarket and intersubmarket link effects.

In reality, the intrasubmarket and the intersubmarket interactions play differing roles in the dynamic process of the new product growth. The intrasubmarket interactions generate local correlations among relatively small groups of consumers that may produce short movements as well as noticeable fluctuations.

Conversely, the intersubmarket interactions form global correlations among all individuals. In other words, the more intense the intersubmarket links, the more dependent the penetration processes in differing submarkets. In the case presented in Figure 3(d), individuals from differing submarkets cannot interact with one another, so the sales in each submarket grow independently. As time passes, the number of "active" submarkets is reduced (because they already exhausted their sales potential one by one), leading to a gradual decline in sales. When intersubmarket interactions exist, the evolution of penetration in any particular submarket is no longer independent of the development of the processes in other submarkets. This phenomenon is demonstrated in Figure 3(e). The formation of a global peak and the diminishing of movements imply that differing submarkets have the tendency to aggregate and become a single unit. The implied practical implication is that product penetration into a specific submarket almost instantaneously accelerates its penetration into other submarkets. A further increase in intersubmarket link intensity yields a convergence of the pattern of penetration of a typical Bass curve.

Figure 3(f) illustrates the case wherein the entire market is considered a single homogeneous unit (i.e., no difference between intrasubmarket and intersubmarket interactions). The intensity of the social ties among all individuals is identical. The probability that during a time period Δt, an individual will interact with and be affected by the WOM of an adopter is $q\Delta t$. That is, $q_s\Delta t = q_w\Delta t \equiv q\Delta t$, as presented in Figure 3(f) for $q\Delta t = 10^{-8}$. Hence, the global WOM coefficient is calculated by Equation (32) to give $Q\Delta t = Mq\Delta t = 10^{-2}$ (note that M is the entire market potential, where $M = 10^6$ and $q\Delta t = 10^{-8}$). The external parameter is defined as $p\Delta t = 10^{-7}$ and so is the global external coefficient. As intuitively expected, the pattern of penetration presented in Figure 3(f) exhibits a Bass-type curve, wherein the time of the peak is measured in exact accordance with the solution of the Bass equation, and is given by

$$t^* = \frac{1}{p\Delta t + Q\Delta t} In\left(\frac{Q\Delta t}{P\Delta t}\right)\Delta t \approx 1{,}150\Delta t, \qquad (33)$$

and the value of the adoption rate at peak is

$$\Delta\overline{m}(t^*) = \frac{M}{4Q\Delta t}(P\Delta t + Q\Delta t)^2 \approx 2{,}500. \qquad (34)$$

Identifying such patterns in real life can help managers take action to transform patterns such as those in Figures 3(a), 3(b), 3(d), and 3(e) into more successful patterns (such as those in 3(c) and 3(f)). Of all six cases, clearly Case C is a rapid, smooth process with a fast takeoff, which offers the highest net present value (NPV) of all. From a managerial point of view, the implication is fairly straightforward: there is probably no need for action. However, there could be a question of efficiency: if marketing investments are high, the firm may be able to increase profits by decreasing these investments. When a rapid takeoff and a smooth process are observed, it is recommended that this option be tested.

In contrast, Case D may be the one with the lowest NPV. Although smooth, because of high WOM activity, the left tail (time to takeoff) is long. Consistent with diffusion literature (e.g., Mahajan et al. 2000), this is an indication for a low external force (p). In such a case, the firm should explore options to increase marketing efforts, or their efficiency in inducing trails. The good news is that in such a case, after takeoff emerges, there is no need to invest further in marketing because the internal force completely takes over.

In the rest of the cases, movements are intrinsic. Case A, although it does not necessarily represent a failure, shows a rapid takeoff that quickly turns into an oscillation-based progression curve. The NPV in this case is relatively low, and the extremely large fluctuations may introduce supply chain management problems. Increasing the external force (p) by marketing efforts can help shift this pattern into the one observed in Case B, which is better for the firm in terms of NPV. If a firm can launch a buzz program focused on weak ties, this case can also be shifted to a faster process.

Cases D and E may be the closest to failures. When the small-scale fluctuations are so frequent (in sales charts we should see something similar to a *thick brush stroke*), this might be an indication for managers to consider "pulling the plug on the product drain." According to research on management failures (Boulding et al. 1997, Biyalogorsky et al. 2006), managers are typically too late in decision making to stop the marketing process of a failure. Perhaps such indication could offer an early signal to consider this option, as this could become a case of losses to the firm.

3.2.1. Conclusions. This study demonstrated the process of movements' emergence in penetration data by means of the intrinsic stochasticity in the dynamics of new product growth. These movements may appear as a consequence of strong local interactions among individuals within social groups. Generally speaking, they last for relatively short time periods. We showed that strengthening the interactions among people from differing social groups can dramatically affect the pattern of penetration by assuring and accelerating the appearance of a global peak of sales. This finding may influence management to enhance the formulation of intersocial relations among adopters and potential adopters who are part of differing social circles. We also demonstrate the strength of the stochastic nature of the process through the occurrence of changes of movements in both *homogeneous* and *stationary* markets. Naturally, in the general case of a heterogeneous market, the appearance of trends becomes even more explicit.

We have shown that by shifting the focus to individual-based data, we can understand the dynamics and behavior of what we used to call noise when using daily or any other granular data. For some products, granular data mean hours, for others it may be weeks; the point is that when we understand the mechanisms and behavior of these movements, we can use them instead of cleaning them. Recent developments in information technology allow managers to get shorter time intervals of data, thereby making the proposed approach more relevant for managers making new product introduction decisions. The most straightforward application can be using the granular data to make predictions. By taking into account that nominal penetration is not a monotonic line, but rather an oscillative one, we can develop a more sophisticated model that can use this pattern instead of smoothing it. This approach appears to be most beneficial when a firm is interested in short-term predictions (e.g., quarters), right after launch, where annual smoothed data do not yet exist.

We now turn to examining empirically how predictions of penetration dynamics can be improved, taking into account that the stochastic emergence of movements is an intrinsic part of the penetration process.

4. Empirical Analysis: Detecting, Tracking, and Forecasting Movements

We started with an analysis of the individual-level based model to come up with a stochastic aggregate-level model. We then demonstrated that penetration movements can be the result of endogenous stochastic processes using an agent-based simulation modeling approach. In this section, we empirically demonstrate how the evolution of current movements can be identified and tracked, and how correct identification can significantly improve predictions. To address this issue, we used a Kalman Filter-based tracker (see, e.g., Blackman 1986, Bar-Shalom and Li 1993, and the Technical Appendix, which can be found at http://mktsci.pubs.informs.org, for more technical details). The apparent advantages of Kalman-Filter estimation of new product diffusion models have already been discussed by Xie et al. (1997), wherein an augmented Kalman filter with continuous state and discrete observations (AKF(C-D)) was implemented to overcome time-interval bias effects, while estimating the dynamics of new product adoption based on differential diffusion models. Kalman filter procedure was also used to handle the case of dynamic brand preferences that are based on a number of time-varying types of product lines (Sriram et al. 2006). As the goal here is to track movements that are stochastically emerged and generally last for relatively short periods of time, we actually focus only on short-term forecasting. Furthermore, as we use granular (daily in this case) data, we may apply quite accurate linear approximations of the dynamic process that standard Kalman-filtering estimation techniques fit well.

The main objective of the proposed tracker is to constantly monitor the data and provide an estimation of the rate mean and its temporal derivative. The use of data sampled at a high frequency together with the intention of conducting a relatively short-term forecast allows use of the Taylor series expansion of the rate mean value within the current trend. Namely, the predicted progression of the averaged penetration rate after a period of time T is approximated by the following equations:

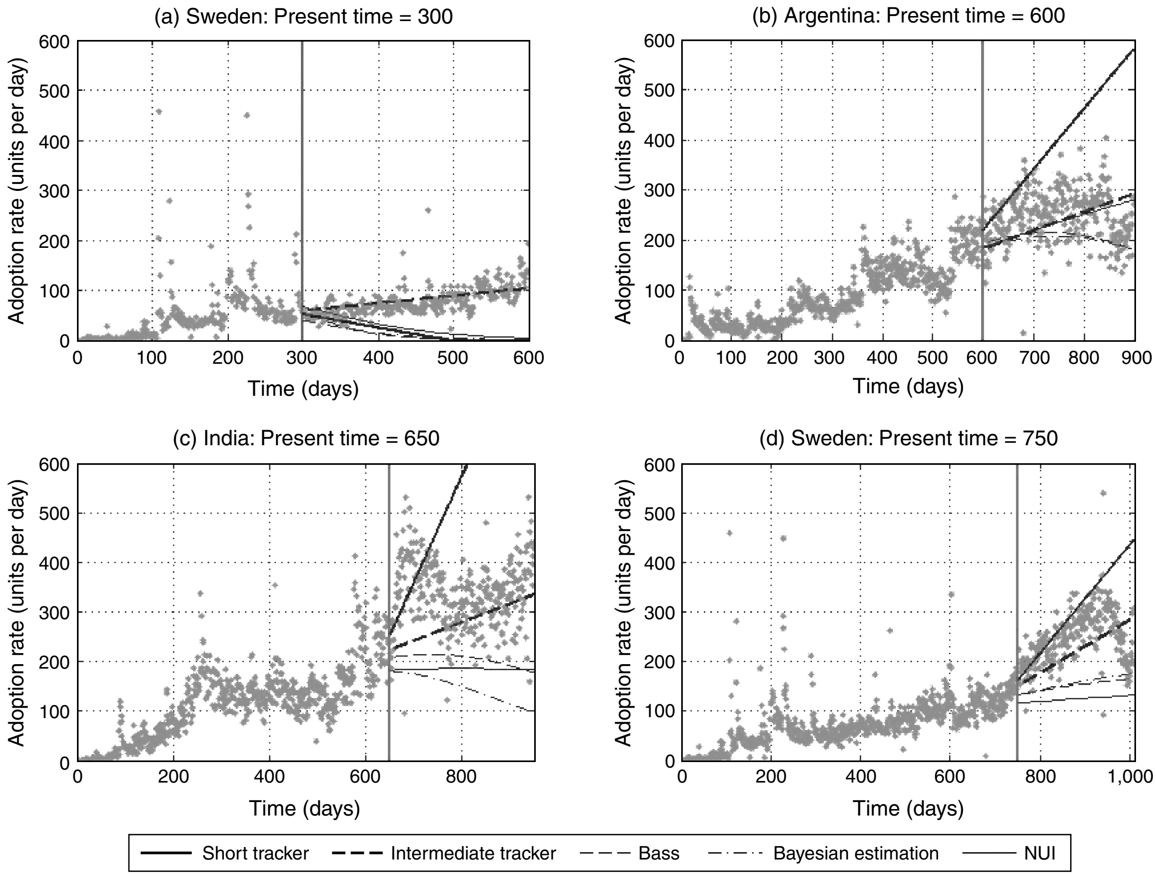
$$x(t+T) \cong x(t) + T\,\dot{x}(t) \tag{35}$$

and

$$\dot{x}(t+T) \cong \dot{x}(t), \tag{36}$$

where $x(t)$ is the penetration rate mean at the time t (i.e., $x(t) \equiv d\bar{m}(t)/dt$ where $\bar{m}(t)$ is the mean value of the cumulative penetration at the time t) and $\dot{x} \equiv dx/dt$. The track state vector, which estimates the values of x and $\dot{x}$, is updated in an adaptive manner by the tracking procedure using the actual noisy penetration measurements alongside the model predictions given by Equations (35) and (36). Note that the origin of the noise in the observed penetration rates is not, or at least not necessarily, rooted in measurement errors. It occurs because of the stochasticity within the penetration process itself, in the sense that the exact values of the actual adoption are not equal to their expected values. Naturally, as long as the new product adoption is developed consistent with a specific trend pattern, the temporal derivative of the penetration rate mean $\dot{x}$ varies slowly. In contrast, a sudden alteration of the current existing trend will generate considerable changes in the values of the penetration rate average and its temporal derivative x and $\dot{x}$, respectively. The tracker is designed to detect and quantify those changes and reinitialize the track state vector, if necessary. Note that unlike the prediction of traditional models (e.g., Bass), changing trends can stochastically create several local peaks in the penetration rate curve. Therefore, a negative sign of the penetration rate mean derivative $\dot{x}$ at present time does not enforce negativity in the future.

Figure 4 Visualization of the Tracker's Forecasting Capability in Four Typical Scenarios



In our empirical validation, we fine-tuned the tracker parameters by applying a trial-and-error search for each one of the data sets. We chose two sets of parameters for each case study. One set is suitable for tracking the movement of a short time scale, while the other set can fit into longer time scale movement estimations, which we define as *intermediate movement tracking*. Sensitivity analysis indicated that the tracker performance is robust to small changes of its system parameters.

In Figure 4, we present four typical scenarios of actual observed daily penetration to depict the forecasting capability of the tracker. A thick vertical line divides each figure into two parts to mark the current present time. Early penetration data are used as a calibration period for future forecasting. The curve starting to the right of the line denotes the future data to be evaluated and is hence unknown for the estimation process.

As already mentioned, each case is examined in two different tracking modes: a short movement tracking mode and an intermediate movement tracking mode. The upcoming predictions of the short and intermediate sale movement trackers are depicted by the thick solid and dashed lines, respectively. We also present the tracker forecasts in comparison with the predictions of three benchmark models described below. The apparent tradeoffs in using differing modes of tracking are depicted in Figure 4.

The short movement tracker is very sensitive to small-scale changes in the current trend. Loss of tracking, therefore, can occur, as can be seen in Figures 4(a) and 4(b). As a result, a short-term (local) decrease in the penetration rate might be interpreted as a final decline in the product life by the short movement tracker (see Figure 4(a)), while sharp short-term variations can distract the track of the penetration out of the primary movement line (see Figure 4(b)). Such undesirable effects can be minimized if an intermediate movement tracker is used that is characterized by a tendency to stay on the path of the primary penetration movement. It may therefore reduce the impact of local peaks on the dynamic evolution of the representative track (see Figure 4(a)) and hold back abrupt variations (see Figure 4(b)). Those restraints, however, can slow the learning process of new trend patterns by the intermediate movement tracker. In contrast, high susceptibility is what allows the short movement tracker to quickly identify and adapt to new trends. These features are well demonstrated in Figures 4(c) and 4(d). The slope of the future increasing penetration is more accurately evaluated by the short movement tracker, while the intermediate tracker succeeds in tracking those rapid trend changes but only "in delay."

Next, we compare the performances of the tracker to three well-known penetration forecasting methods serving as benchmarks for goodness-of-fit measures of the forecasts. The motivation for this comparison is neither to evaluate nor to argue for or against these methods. All three methods were developed with annual data and smoothing strategy in mind, and their goal is not short-term prediction at early stages, for which our proposed model was developed. However, because these methods are foundations of penetration forecasting, they can serve as benchmark models for this approach on its own terms. The absence of other models developed for this particular purpose also led us to select the best-known models. The three models selected as benchmarks are therefore as follows:

(1) **The Bass model.** A nonlinear least-squares (NLS) estimation using past penetration data to obtain the parameters of the Bass model according to the procedure suggested by Srinivasan and Mason (1986). Accordingly, a straightforward extrapolation of future penetration was conducted commensurate with the solution of the Bass equation. The Bass benchmark is displayed by a thin dashed curve in each of the following figures.

(2) **The nonuniform influence model.** The nonuniform influence (NUI) model was introduced by Easingwood et al. (1983) and is considered to be an advanced and more realistic one. This model takes into account the fact that the WOM effect does not remain constant over the entire diffusion process and enables the appearance of asymmetrical diffusion patterns around the stage of maximum adoption rate. In this benchmark, we used an NLS estimation using past data to evaluate the parameters of the NUI model, which we used to predict the penetration in the future. The NUI benchmark forecasts are displayed by thin solid curves in each of the following figures.

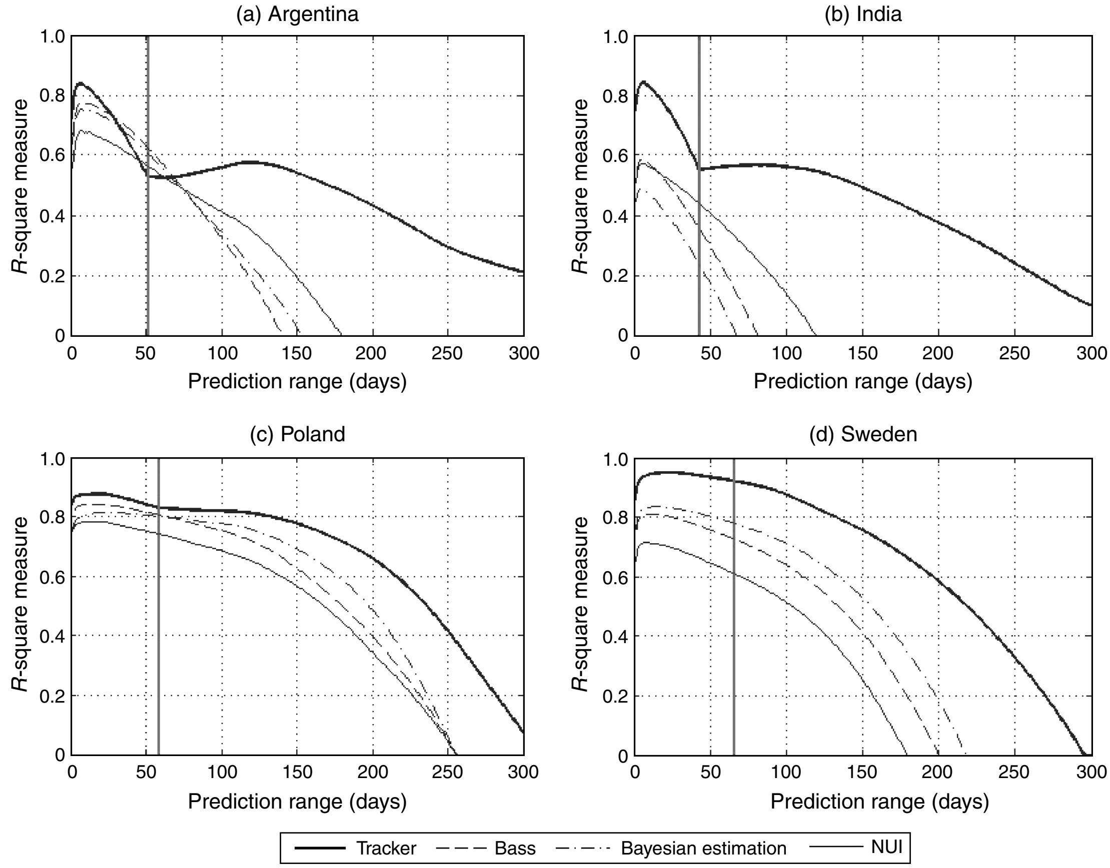
(3) **A learning Bayesian estimation.** Here, we have applied the maximum a posteriori estimator via Bayes' formula to extract the penetration coefficients from past penetration data following Lenk and Rao (1990). Those coefficients were later used for future penetration forecasting. Rather than evaluating the prior distributions of the Bass model parameters across various products (see Lenk and Rao 1990), we obtained the prior distributions through a prior estimation of the past penetration data itself, using the confidence intervals for the coefficients of the fit results, which were assumed to be normally distributed. The posterior distribution of the observations was assumed to be normal with means given by Bass model interpolations and with known variances at any given time. Those variances are derived from the standard deviation estimators of our high-frequency sampled data, as we use daily data. Given that we use the penetration information itself, the prediction should be more accurate than Lenk and Rao's (1990) model, which does not assume the existence of such information. The learning Bayesian-benchmark forecasts are displayed by a thin dash-dot line in each figure.

Comparing the forecasts of the proposed adaptive movement tracker with the predictions of the three benchmark models, it appears that changes in the penetration pattern are difficult to follow on the basis of a strict Bass or NUI modeling. This difficulty is mainly attributed to the restriction of having only one maximum in the penetration curve. Thus, an observed (local) peak in past data (see Figures 4(a) and 4(c)) or a curvature alteration of the primary movement from a convex to a concave form (see Figures 4(b) and 4(d)) may be interpreted as reaching the area of the single maximum. As a result, forecasts in such cases tend to underestimate future penetration. The tracker, by contrast, is designed to adopt the dynamic changes in the current trend by using a dynamic weighting procedure of the incorporated observations in its representative track, thereby providing better forecasting in such cases.

Figures 5 and 6 provide a quantitative evaluation of the proposed movement tracker in its two tracking modes as compared to the predictions of our three benchmarks. Figure 5 presents the four cases of penetration of the e-mail software tool (presented in the introduction) in four markets: Argentina, India, Poland, and Sweden. Figure 6 presents the performance evaluation of the tracker vis-à-vis the three benchmarks using three additional data sets of movie rental records. The data were collected between October 1998 and December 2005 from approximately 480,000 customers in a daily resolution.

Figures 5 and 6 display an R-square measure for the goodness of fit of the cumulative predictions of the models (the tracker and the benchmarks) as a function of the prediction range. For example, $R^2(100)$ rates the ability of a specific forecasting model (e.g., Tracker, Bass, or NUI) to predict the total depth of penetration over the coming 100 days. The R-square measure may be referred to as the proportion of variation explained by the model. Thus the closer the value of the R-square measure to one, the better the forecasts of the model. The vertical line around a prediction range of 50 days in each case stands for the maximal range wherein the short movement tracker predictions are better than those of the intermediate movement tracker. The solid R-square curve in each case

Figure 5 The *R*-Square Measures for the Goodness of Fit of the Cumulative Predictions of the Models (the Tracker and the Three Benchmarks) as a Function of the Prediction Range for the Four Case Studies: (a) Argentina, (b) India, (c) Poland, and (d) Sweden



designates the goodness of fit of the forecasts produced by the short (intermediate) movement tracker. The thin dashed and dash-dotted *R*-square curves describe the goodness of fit of the Bass model predictions and Bayesian estimation procedures, respectively. The thin solid *R*-square curve designates the goodness of fit of the NUI model predictions.

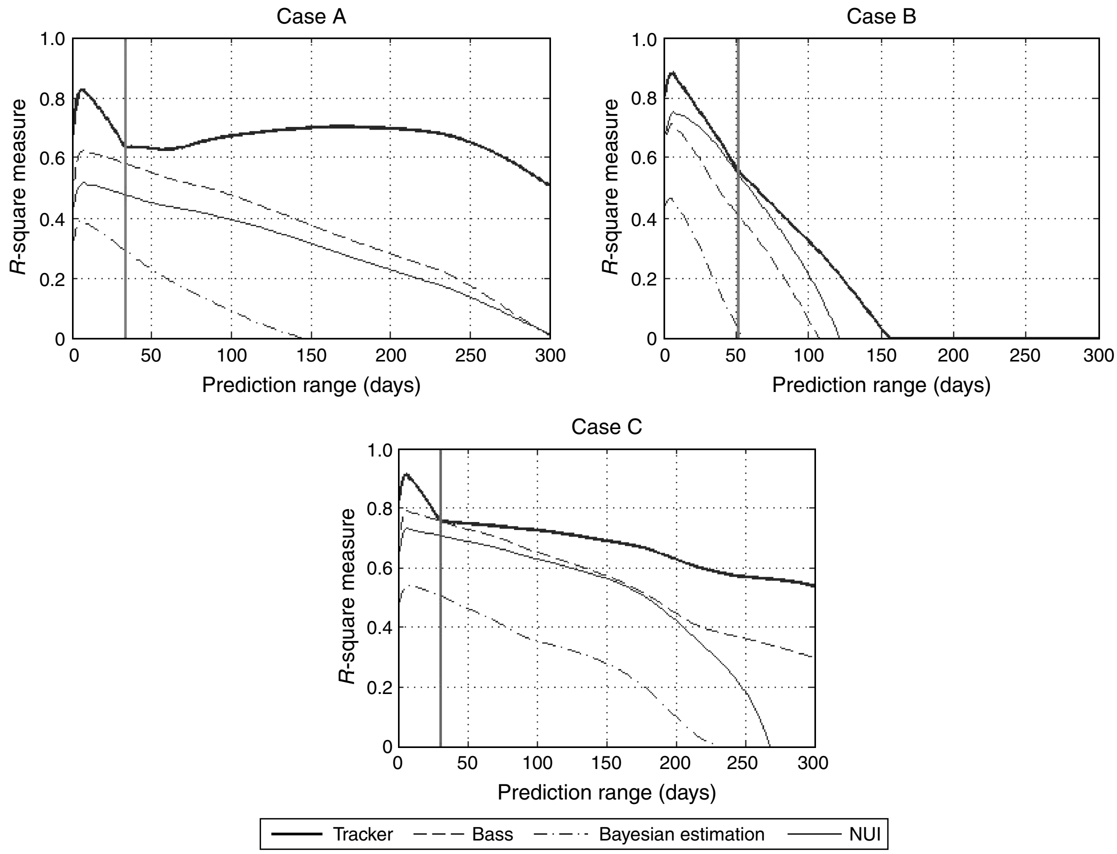
The results indicate that the tracker in both its tracking modes exhibits superior future cumulative forecasts. With the exception of the Argentinean market of the e-mail software tool during a relatively small range of 30 to 70 days (see Figure 5(a)), the tracker predictions of future cumulative penetration are substantially more accurate in all prediction ranges (up to 300 days) in all seven cases. These results become stronger following the drop in the values of the *R*-square measures of the Bass and the NUI models' predictions in intermediate ranges.

In general, the tracker can provide good forecasting results in ranges between 100 and 200 days. For the e-mail software tool data sets, the tracker provides *R*-square measures in a prediction range of 200 days of about 0.4 in the cases of Argentina and India and 0.6 in the cases of Poland and Sweden, while the measures of the Bass and the NUI approximations drop to zero in all the markets.

In the first and the third data sets of movie rental records (see Figures 6(a) and 6(c)), the tracker carries out predictions with *R*-square measures of 0.7 and 0.6, respectively, in the 200-day range. In contrast, the benchmarks (NLS-Bass and NUI) produce *R*-square values of about 0.3 in the first and the third movie rental record data sets. In the second movie rental record data set, the *R*-square measure of all the models drops to zero in a 200-day prediction range, yet the tracker still achieves the better forecasting results.

Interestingly, although the NUI model fits better than the Bass model with past data, it does not exhibit better predictions in some of the cases. Again, this may be because of the impact of the changing movements in the pattern-of-adoption rate. That is, the estimated parameters of the models (Bass and NUI) provide averaged values, but essentially these parameters are not constant when we zoom in to high sampling resolution (e.g., daily) as movements in penetration exist. Moreover, because both the Bass and NUI models denote growth processes, their predictions are very sensitive to deviations and biases in their parameters'

Figure 6 The *R*-Square Measures for the Goodness of Fit of the Cumulative Predictions of the Models (the Tracker and the Three Benchmarks) as a Function of the Prediction Range for the Three Data Sets of Movie Rental Records



values. The NUI model appears more sensitive to movements when using granular data because it takes into account changes in internal influence over time. In contrast, the tracker is almost by definition more tolerant of and adaptive to dealing with changing trends. Apparently, when zooming in to granular data, either much more sensitivity to movements or a complete ignorance is needed.

5. Discussion

The main premise of this work is that the dynamics of the fluctuations in penetration curves can be informative. This study deals with two important issues occasionally faced in innovation management: identifying penetration patterns (i.e., distinguishing between movements and fluctuations) and forecasting future sales in early stages of product life.

By zooming in on the individual consumer and on the time periods (i.e., daily sales data and early part of the product life cycle), it is possible to add meaningful information for analysis. We propose a universal modeling platform that can capture the stochastic nature of sales dynamics that at the same time allows for aggregating individual behavior. To this end, granular data are used to come up with a short time (i.e., daily), individual-level model. Stepping into the noise inherent in short time data enables the emergence of movements that can be understood by considering this noise as meaningful information.

Overall, we demonstrate that differing sales patterns can exist even when the market is composed of identical consumers, and that these differences are embedded in the various inter- and intragroup social ties. We do not argue that exogenous events do not occur or cannot cause sales movements. However, we show that even without external events, these movements occur quite often. We also argue that this frequent occurrence is in fact the natural and standard penetration pattern when looking at granular data. By taking this phenomenon into account, better predictions can be made.

Based on the concept of focusing on daily sales data, a sales tracker was developed and shown to provide fairly accurate predictions even at early stages of the penetration process, whereas models that use more coarse-grained data cannot operate. More precisely, the current approach enables us to obtain predictions of future growth just a short time after launching a new product. For long-term predictions, at a later stage, this approach does not provide any advantage over other standard methods. Predicting future short-term sales, however, has important managerial implications, as it can provide better management of production and inventories such that potential lost sales or excess inventories will be minimized. Such prediction capabilities are particularly important in light of the ever-shorter product life cycle in many industries, e.g., personal computers, wherein the life cycle of many components is between one and two years (e.g., Kurawarwala and Matsu 1996).[3] Furthermore, other industries, such as services, are leaning toward personalized interactions (Rust and Chung 2006) and therefore stressing the need for individual (granular)-based models.

An important question is how robust the model is to errors resulting from data that are not at the industry level. While the first study used penetration data of a unique product, and the data can be considered a good proxy for an industry level, the second study used a clear case of partial data (firm level). In addition, the data we have contain only a subset of all the data. The results indicate that despite such a limitation, the proposed model works well, and its predictions are relatively high.

The proposed framework and modeling approach lay the groundwork for further research. Extending the current model to account for repeat purchase goods is a natural step that can provide meaningful insight into the emergence of sales trends. Another avenue for future research might focus on identification of opinion leaders and their effect on other consumers through social ties.

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Appendix. The Bass Model as a Private Case of the Proposed Universal Framework
In this appendix we use the proposed universal framework to derive the well-known Bass model as a special case.

Consider the case in which only a person who had already adopted the innovation can apply word of mouth on his or her environment. We substitute the transition probability for any individual i given in Equation (3) in Equation (11), the net market force, to obtain

$$F(1,\Omega_t)\approx\sum_{i=1}^{M}(1-s_i(t))\left(p_i(t)+\sum_{\substack{j=1\\ j\neq 1}}^{M}w_{ij}(t)s_j(t)\right), \tag{37}$$

where $p_i(t)$ is the probability per unit of time that potential adopter i will adopt the innovation because of external influence (e.g., advertising); $w_{ij}(t)$ is the probability per unit of time that an individual i will purchase the innovation as result of the WOM communicated by individual j, who has already adopted the innovation; and $s_i(t)$ is the ith individual index of consumption.

Let us further assume that the intensity of the WOM communications among individuals as well as the external influence on potential adopters does not vary with time and is identical for all consumers. That is, the market is considered to be stationary and homogeneous. In that case, the net market force can be rewritten as

$$F(1,\Omega_t)\approx\sum_{i=1}^{M}(1-s_i(t))\left(p+w\sum_{\substack{j=1\\ j\neq 1}}^{M}s_j(t)\right), \tag{38}$$

where p denotes the probability per unit of time that a customer will be persuaded by an external influence to adopt the new product, and w expresses the probability per unit of time that a potential adopter will interact with an actual adopter of the innovation and will also be affected by his or her WOM. We define

$$m(t)=\sum_{i=1}^{M}s_i(t) \tag{39}$$

as the total cumulative sales. Then applying the approximation $\sum_{\substack{j=1\\ j\neq 1}}^{M}s_j(t)\approx m(t)$, Equation (38) takes the form

$$F(t,\Omega_t)\approx\hat{F}(m(t))=(M-m(t))\left(p+\frac{Q}{M}m(t)\right), \tag{40}$$

where $P=p$ and $Q=Mw$ are the macroscopic coefficients of the external and the internal influence, respectively. In other words, we approximate the interaction among individuals via the mean of the consumption indexes $m(t)/M$. As can be seen, the partial information on which our model of the net market force is based on is the total cumulative sales, namely, $\widehat{\Omega}_t=m(t)$.

After modeling the net market force, we can now use Equation (12) of the sales motion to describe the dynamics of sales for new product sales, as follows:

$$\Delta m(t)=\left((M-m(t))\left(p+\frac{Q}{M}m(t)\right)\right)\Delta t+u(t). \tag{41}$$

As a special case, we can set Equation (22) to the continuous limit, while neglecting the stochastic effects of the process to obtain the following ordinary differential equation,

$$\frac{dm(t)}{dt}=(M-m(t))\left(p+\frac{Q}{M}m(t)\right), \tag{42}$$

which is the Bass equation (Bass 1969).

[3] Other early frameworks for prediction are offered by the literature. For example, Garber et al. (2004) used spatial density of sales as additional information to make a significant contribution to the early prediction of product success. According to this approach, a measure of the distance of the sales density from a uniform function is highly correlated with success. However, the dependent variable using this approach was a dichotomy of success versus failure, and there was no indication as to what the sales curves in the next periods would look like.

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