JUNE 2005 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. Required To Calculate: \( 4 \frac{1}{5} - \left( 1 \frac{1}{9} \times 3 \right) \)

Calculation:

\[
\begin{align*}
4 \frac{1}{5} - \left( 1 \frac{1}{9} \times 3 \right) &= 4 \frac{1}{5} - \left( \frac{10}{9} \times 3 \right) \\
&= 4 \frac{1}{5} - \frac{10}{3} \\
&= 4 \frac{1}{5} - 3 \frac{1}{3} \\
&= \frac{21}{5} - \frac{10}{3} \\
&= \frac{3(21) - 5(10)}{15} \\
&= \frac{63 - 50}{15} \\
&= \frac{13}{15} \\
&= \frac{13}{15} \text{ (in exact form)}
\end{align*}
\]

b. Data: Table showing Amanda’s shopping bill

(i) Required To Calculate: The values of \( A, B, C \) and \( D \)

Calculation:

3 T-shirts at $12.50 each cost a total of

\[3 \times 12.50 = 37.50\]

\[\therefore A = 37.50\]

2 CD’s cost a total of $33.90

\[\therefore \text{The unit price is } \frac{33.90}{2} = 16.95\]

\[\therefore B = 16.95\]
C posters at $6.20 each cost $31.00

\[ C = \frac{31.00}{6.00} \]

\[ C = 5 \]

The total bill is $108.28

\[ 15\% \text{ VAT} = \frac{15}{100} \times 108.28 \]

\[ = 16.242 \]

\[ = 16.24 \text{ to the nearest cent} \]

\[ D = 16.24 \]

(ii) **Required To Determine:** Whether Amanda made a profit or a loss

**Solution:**

Price paid for 6 stickers at $0.75 each and 6 stickers at $0.40 each

\[ = (6 \times 0.75) + (6 \times 0.40) \]

\[ = 4.50 + 2.40 \]

\[ = 6.90 \]

The cost of 12 stickers to Amanda = $5.88

Since the selling price > Cost price, then Amanda acquired a profit of

\[ (6.90 - 5.88) \]

\[ = 1.02 \]

2. **a. Required To Factorise:** (i) \(5a^2b + ab^2\), (ii) \(9k^2 - 1\), (iii) \(2y^2 - 5y + 2\)

**Factorising:**

(i) \(5a^2b + ab^2\)

\[ = 5.a.a.b + a.b.b \]

\[ = ab(5a + b) \]

(ii) \(9k^2 - 1\)

\[ = (3k)^2 - (1)^2 \]

This is the difference of two squares

\[ (3k-1)(3k+1) \]

(iii) \(2y^2 - 5y + 2\)

\[ = (2y-1)(y-2) \]
b. **Required To Simplify:** \((2x + 5)(3x - 4)\)

**Solution:**

Simplifying \((2x + 5)(3x - 4)\)

\[
= 6x^2 + 15x - 8x - 20
\]

\[
= 6x^2 + 7x - 20
\]

c. **Data:** Card game played among 3 people.

**Solution:**

Score by Adam = \(x\) points

Imran’s score is 3 less than Adam’s score = \((x - 3)\) (data)

(i) **Required To Find:** an expression in terms of \(x\) for the number of points scored by Shakeel.

**Solution:**

Shakeel’s score is 2 times Imran’s score = \(2(x - 3)\) points

(ii) **Required To Find:** an equation which may be used to find the value of \(x\).

**Solution:**

Total score = 39 points

\[
\therefore x + (x - 3) + 2(x - 3) = 39
\]

\[
x + x - 3 + 2x - 6 = 39
\]

\[
4x - 9 = 39
\]

\[
4x = 48
\]

and \(x = 12\)

3. a. **Data:** Venn diagram illustrating the students in a class who study Music and /or Dance.

**Solution:**

![Venn Diagram]

(i) **Required To Calculate:** the number of students who take both Music and Drama.

**Calculation:**

\(n(M) = 24\) (data)
\[3x + x = 24\]
\[4x = 24\]
\[x = 6\]

And \(n(M \cap D),\) that is number of students who take both Music and Dance \(= 6\)

(ii) **Required To Calculate:** the number of students who take Drama only.

**Calculation:**

Hence

\[\frac{D}{n(D \text{ only})} = 13\]

That is, the number of students who take Dance only \(= 13\)

b. **Data:** Line with gradient \(\frac{2}{3}\) passes through \(P(-3, 5)\)

(i) **Required To Find:** the equation of the line through \(P(-3, 5)\) and with gradient \(\frac{2}{3}\).

**Solution:**

Equation of line is

\[\frac{y - 5}{x - (-3)} = \frac{2}{3}\]
\[3y - 15 = 2x + 6\]
\[3y = 2x + 21\]
\[y = \frac{2}{3}x + 7\]

is of the form \(y = mx + c\), where \(m = \frac{2}{3}\) and \(c = 7\).
(ii) **Required To Prove:** the above line is parallel to the line $2x - 3y = 0$

**Solution:**

$2x - 3y = 0$

$3y = 2x$

$y = \frac{2}{3}x$

is of the form $y = mx + c$, where $m = \frac{2}{3}$ is the gradient.

Hence $y = \frac{2}{3}x + 7$ and $2x - 3y = 0$ are parallel since they both have the same gradient $\left( \frac{2}{3} \right)$ and parallel lines have the same gradient.

4. **Data:** Diagrams of

![Diagram of small and medium pizzas]

**a. Required To Determine:** Whether a medium pizza is twice as large as a small pizza.

**Solution:**

The pizzas are 3-dimensional, hence a comparison of sizes must be made by comparing their volumes. Both have the same height (thickness).

![Diagram of small and medium pizza volumes]

Volume of small pizza

$V_s = \pi r^2 h$

$= \pi (7.5)^2 h$

$= 56.25 \pi h \text{ cm}^3$

Volume of medium pizza

$V_m = \pi r^2 h$

$= \pi (15)^2 h$

$= 225 \pi h \text{ cm}^3$

So we see that the medium pizza has 4 times the volume of a small pizza. So that statement – A medium pizza is twice as large as a small pizza is **INCORRECT**.
b. **Data:** The prices for each slice of a medium pizza and for one small pizza.  
**Required To Find:** Whether it is better to buy 1 medium pizza or 4 small pizzas.  
**Solution:**

\[
\text{Cost of } \frac{1}{3} \text{ medium pizza} = 15.95\\
\therefore \text{Cost of an entire medium pizza} = 15.95 \times 3 = 47.85
\]

Cost of 1 small pizza = 12.95

Since 4 small pizzas \(\equiv\) 1 medium pizza

Then the equivalent cost of 4 small pizzas

\[= 12.95 \times 4\]

\[= 51.80\]

4 small pizzas is equivalent in volume to 1 medium pizza which costs 47.95

\[
\therefore \text{The ‘better buy’ (which supposedly means more pizza at a lesser price) is obtained by buying a medium pizza.}
\]

5. **a. Data:** The coordinates of the vertices of a triangle, \(D, E\) and \(F\).  
**Required To Draw:** \(\Delta DEF\).  
**Solution:**

\[D(1, 5) \quad E(4, 7) \quad F(D) \]

b. **(i) Required to draw:** \(\Delta D'E'F'\)  
**Solution:**

\[D' = (7, 1) \quad E' = (5, 1) \quad F' = (7, 4)\]
(ii) Required To Draw: $\Delta D''E''F''$
Solution:

$D'' = (7, -4) \quad E'' = (5, -4) \quad F'' = (7, -1)$

(iii) Required to identify: the type of transformation that maps $\Delta DEF$ onto $\Delta D''E''F''$

Solution:

$\Delta DEF \quad \text{Reflection in } x = 4 \rightarrow \Delta D'F'E' \quad \text{Translation} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \rightarrow \Delta D''E''F''$

Hence $\Delta DEF \quad \text{Glide reflection} \rightarrow \Delta D''E''F''$
c. **Data:** Stick 1.8 m casts a shadow 2 m long.

![Diagram of sunlight casting a shadow](image)

**Required To Calculate:** Angle of elevation of the sun  
**Calculation:**  
Let the angle of elevation be \( \alpha \)  
\[
\tan \alpha = \frac{1.8}{2} 
\]
\[
\alpha = \tan^{-1}(0.9) 
\]
\[
\alpha = 41.9^\circ 
\]
\[
\alpha = 42^\circ \text{ (to the nearest degree)} 
\]

6. a. **Data:** Diagram of a pentagon ABCDE  
**Required To Calculate:** \( x^\circ, y^\circ \)  
**Calculation:**  
(i) \( \hat{E}AD = 57^\circ \) (alternate angles)  
\[
x^\circ = 180^\circ - (80^\circ + 57^\circ) 
\]
\[
= 43^\circ 
\]
(Sum of angles in a triangle = 180°)

(ii) \[ \hat{B}AD = 108° - 57° \]
\[ = 51° \]
\[ \therefore y = 360° - (51° + 80° + 57°) \]
\[ = 162° \]
(Sum of angles in a quadrilateral = 360°)

b. Data: \( f(x) = \frac{1}{2}x + 5 \quad g(x) = x^2 \)

(i) Required To Evaluate: \( g(3) + g(-3) \)
Solution:
\[ g(3) + g(-3) \]
\[ = (3)^2 + (-3)^2 \]
\[ = 9 + 9 \]
\[ = 18 \]

(ii) Required To Evaluate: \( f^{-1}(6) \)
Solution:
Let \( y = \frac{1}{2}x + 5 \)
\[ y - 5 = \frac{1}{2}x \]
\[ 2y - 10 = x \]
Replace \( y \) by \( x \)
\[ f^{-1}(x) = 2x - 10 \]
\[ \therefore f^{-1}(6) = 2(6) - 10 \]
\[ = 2 \]

(iii) Required To Evaluate: \( fg(2) \)
Solution:
\[ g(2) = (2)^2 \]
\[ = 4 \]
\[ fg(2) = f(4) \]
\[ = \frac{1}{2}(4) + 5 \]
\[ = 2 + 5 \]
\[ = 7 \]
7. **Data:** Table showing the height of 400 applicants for the police service.

   a. **Required To Draw:** the cumulative frequency curve of heights given in the table.

   **Solution:**
   The data shows a Continuous variable and we create the table as:

<table>
<thead>
<tr>
<th>Height in cm, ( x )</th>
<th>L.C.B.</th>
<th>U.C.B</th>
<th>No. of applicants</th>
<th>Cumulative frequency</th>
<th>Points to be plotted (U.C.B, CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>151 – 155</td>
<td>150.5</td>
<td>155.5</td>
<td>10</td>
<td>10</td>
<td>(150.5, 0) ( \leq x &lt; 155.5 )</td>
</tr>
<tr>
<td>156 – 160</td>
<td>155.5</td>
<td>160.5</td>
<td>55</td>
<td>65</td>
<td>(160.5, 65)</td>
</tr>
<tr>
<td>161 – 165</td>
<td>160.5</td>
<td>165.5</td>
<td>105</td>
<td>170</td>
<td>(165.5, 170)</td>
</tr>
<tr>
<td>166 – 170</td>
<td>165.5</td>
<td>170.5</td>
<td>110</td>
<td>280</td>
<td>(170.5, 280)</td>
</tr>
<tr>
<td>171 – 175</td>
<td>170.5</td>
<td>175.5</td>
<td>80</td>
<td>360</td>
<td>(175.5, 360)</td>
</tr>
<tr>
<td>176 – 180</td>
<td>175.5</td>
<td>180.5</td>
<td>30</td>
<td>390</td>
<td>(180.5, 390)</td>
</tr>
<tr>
<td>181 – 185</td>
<td>180.5</td>
<td>185.5</td>
<td>10</td>
<td>400</td>
<td>(185.5, 400)</td>
</tr>
</tbody>
</table>

\[ \sum f = 400 \]

The point \((150.5, 0)\) is obtained by extrapolation, so as to start the curve on the horizontal axis.
b. (i) **Required To Estimate:** the number of applicants whose heights are less than 170 cm.
**Solution:**
From the graph, \( \approx 265 \) applicants are less than 170 cm (read off).

(ii) **Required to estimate:** the median height of applicants.
**Solution:**
The median height of applicants \( \approx 167 \) cm (read off).

(iii) **Required to estimate:** the height that 25% of the applicants are less than
**Solution:**
\[
25\% \text{ of the applicants} = \frac{25}{100} \times 400 = \frac{25}{100} \times 400 = 100
\]
100 applicants are less than 162 cm (read off).
(iv) **Required To Estimate:** the probability that a randomly selected applicant has a height no more than 162 cm.

**Solution:**

\[ P(\text{applicant's height is no more than 162 cm}) = \frac{\text{No. of applicants} \leq 162 \text{ cm}}{\text{No. of applicants}} \]

\[ = \frac{100}{400} \]

\[ = \frac{1}{4} \]

8. a. **Data:** Table showing a number pattern

**Required To Complete:** the table given.

**Solution:**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^3)</td>
<td>((0 \times 3^2) + (3 \times 2) + 2)</td>
<td>8</td>
</tr>
<tr>
<td>(3^3)</td>
<td>((1 \times 4^2) + (3 \times 3) + 2)</td>
<td>27</td>
</tr>
<tr>
<td>(4^3)</td>
<td>((2 \times 5^2) + (3 \times 4) + 2)</td>
<td>64</td>
</tr>
<tr>
<td>(5^3)</td>
<td>((3 \times 6^2) + (3 \times 5) + 2)</td>
<td>125</td>
</tr>
</tbody>
</table>

- 2 less than the number that is being cubed.
- 1 more than the number that is being cubed.
- The number that is being cubed.
- The result.

\[(i)\] \(6^3\) \[= (6 - 2) \times (6 + 1)^2 + (3 \times 6) + 2\]
\[= (4 \times 7^2) + (3 \times 6) + 2\]
\[= 216\]

\[(ii)\] \(10^3\) \[= (10 - 2) \times (10 + 1)^2 + (3 \times 10) + 2\]
\[= (8 \times 11^2) + (3 \times 10) + 2\]
\[= 1000\]

\[(iii)\] \(n^3\) \[= (n - 2) \times (n + 1)^2 + (3 \times n) + 2\]
\[= n^3\]
b. Required to prove: \((a - b)^3(a + b) + ab(a + b) = a^3 + b^3\)

Proof: L.H.S.

\[(a - b)^3(a + b) + ab(a + b)\]
\[= (a^2 - 2ab + b^2)(a + b) + ab(a + b)\]
\[= a^3 - 2a^2b + ab^2 + a^2b - 2ab^2 + b^3 + a^2b + ab^2\]
\[= a^3 + b^3\]
\[= \text{R.H.S.}\]

Q.E.D.

Section II

9. a. Required to express: \(5x^2 + 2x - 7\) in the form \(a(x + b)^2 + c\), \(a,b,c \in \mathbb{R}\)

Solution:

\[a(x + b)^2 + c\]
\[= a(x^2 + 2bx + b^2) + c\]
\[= ax^2 + 2abx + ab^2 + c\]

Equating the coefficient of \(x^2\)
\[a = 5 \in \mathbb{R}\]

Equating the coefficient of \(x\)
\[2(5)b = 2\]
\[b = \frac{1}{5} \in \mathbb{R}\]

Equating constants
\[5\left(\frac{1}{5}\right)^2 + c = -7\]
\[\frac{1}{5} + c = -7\]
\[c = -7\frac{1}{5} \in \mathbb{R}\]

\[\therefore 5x^2 + 2x - 7 = 5\left(x + \frac{1}{5}\right)^2 - 7\frac{1}{5}\]

OR

\[5x^2 + 2x - 7\]
\[= 5\left(x^2 + \frac{2}{5}x\right) - 7\]
Half the coefficient of \( x \) is \( \frac{1}{2} \left( \frac{2}{5} \right) = \frac{1}{5} \)

\[
= 5 \left( x^2 + \frac{2}{5} x + \frac{1}{25} \right)
\]

\[
= 5x^2 + 2x + \frac{1}{5}
\]

\[
- 7 \frac{1}{5}
\]

\[
- 7
\]

\[
= 5 \left( x + \frac{1}{5} \right)^2 - 7 \frac{1}{5}
\]

is of the form \( a(x + b)^2 + c \), where

\( a = 5 \in \mathbb{R} \)

\( b = \frac{1}{5} \in \mathbb{R} \)

\( c = -7 \frac{1}{5} \in \mathbb{R} \)

b. (i) **Required To Determine:** the minimum value of \( y = 5x^2 + 2x - 7 \)

**Solution:**

\[
y = 5x^2 + 2x - 7
\]

and

\[
y = 5 \left( x + \frac{1}{5} \right)^2 - 7 \frac{1}{5}
\]

\[
5 \left( x + \frac{1}{5} \right)^2 \geq 0 \quad \forall x
\]

\[
\therefore \ y_{\min} = 0 - 7 \frac{1}{5}
\]

\[
= -7 \frac{1}{5}
\]
(ii) **Required To Determine:** the value of $x$ at which the minimum point occurs.

**Solution:**

When

$$5\left(x + \frac{1}{5}\right)^2 = 0$$

$$\left(x + \frac{1}{5}\right)^2 = 0$$

$$x + \frac{1}{5} = 0$$

$$x = -\frac{1}{5}$$

**OR**

$$y = 5x^2 + 2x - 7$$ has an axis of symmetry at

$$x = \frac{-2}{2(5)}$$

$$= -\frac{1}{5}$$

At minimum point $x = -\frac{1}{5}$ and $y = 5\left(-\frac{1}{5}\right)^2 + 2\left(-\frac{1}{5}\right) - 7$

$$= -7\frac{1}{5}$$

$y_{\text{min}} = -7\frac{1}{5}$ at $x = -\frac{1}{5}$
c. **Required To Solve**: \(5x^2 + 2x - 7 = 0\)

**Solution:**

\[5x^2 + 2x - 7 = 0\]
\[(5x + 7)(x - 1) = 0\]
\[\therefore x = 1 \text{ or } -\frac{7}{5}\]

OR

\[x = \frac{-2 \pm \sqrt{(2)^2 - 4(5)(-7)}}{2(5)}\]
\[= \frac{-2 \pm \sqrt{4 + 140}}{10}\]
\[= \frac{-2 \pm \sqrt{144}}{10}\]
\[= \frac{-2 \pm 12}{10}\]
\[= -\frac{14}{10} \text{ or } 10\]
\[= -1 \frac{2}{5} \text{ or } 1\]

OR

\[5x^2 + 2x - 7 = 0\]
\[5\left(x + \frac{1}{5}\right)^2 - 7 \cdot \frac{1}{5} = 0\]
\[5\left(x + \frac{1}{5}\right)^2 = \frac{36}{5}\]
\[\left(x + \frac{1}{5}\right)^2 = \frac{36}{25}\]

Find the square root
\[\left(x + \frac{1}{5}\right) = \pm \frac{6}{5}\]
\[x = -\frac{1}{5} \pm \frac{6}{5}\]
\[= -\frac{1}{5} \pm \frac{6}{5}\]
\[= -\frac{7}{5} \text{ or } 1\]
c. **Required To Sketch:** the graph of \( y = 5x^2 + 2x - 7 \), showing the coordinates of the minimum point, the value of the \( y \)– intercept and the points where the graph cuts the \( x \)– axis.

**Solution:**

When \( x = 0 \)

\[ y = 5(0)^2 + 2(0) - 7 = -7 \]

\[ \therefore \text{Curve cuts the } y\text{–axis at (0, -7) and the } x\text{–axis at 1 and } -\frac{7}{5}. \]

Minimum point = \( \left( -\frac{1}{5}, -\frac{7}{5} \right) \)

![Graph of \( y = 5x^2 + 2x - 7 \)]

10. a. **Data:** Speed – time graph for the movement of a cyclist.

(i) **Required To Calculate:** The acceleration of the cyclist during the first 15 seconds.

**Calculation:**

\[
\text{Gradient} = \frac{40 - 0}{15 - 0} = \frac{2}{3} \]

\[ \therefore \text{Acceleration} = \frac{2}{3} \text{ms}^{-2} \]
(ii) **Required To Calculate:** The distance travelled by the cyclist between \( t = 15 \) and \( t = 35 \).

**Calculation:**

The distance covered between \( t = 15 \) and \( t = 35 \) is the area of the region, \( A \), shown in the diagram which describes a trapezium.

\[
A = \frac{1}{2}(40 + 50) \times (35 - 15)
\]

\[
= 900 \text{ m}
\]

b. **Data:** Diagram showing the distance – time journey of an athlete

(i) **Required To Calculate:** The average speed during the first 2 hours.

**Calculation:**

The average speed during the first 2 hours

\[
= \frac{\text{Total distance covered}}{\text{Total time taken}}
\]

\[
= \frac{12 \text{ km}}{2 \text{ h}}
\]

\[
= 6 \text{ kmh}^{-1}
\]

(ii) **Required To Determine:** What the athlete did between 2 and 3 hours after the start of the journey.

**Solution:**
At \( t = 2 \), distance = 12 km and at \( t = 3 \), distance = 12 km. This is indicated by a horizontal branch in the graph. Hence, between 2 and 3 hours after the start, the cyclist did NOT travel OR the cyclist stopped cycling for that 1 hour interval.

(iii) **Required To Calculate:** the average speed on the return journey.

**Solution:**

The return journey took \( 5 - 3 \frac{1}{2} \) hours.

\[
\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}
\]

\[
= \frac{12 \text{ km}}{1 \frac{1}{2} \text{ h}}
\]

\[
= 8 \text{ kmh}^{-1}
\]

(c) **Data:** Diagram of a triangle bounded by lines \( GH \), \( GK \) and \( HK \).

(i) **Required To Find:** the equation of the line \( HK \)

**Solution:**
HK is a vertical line that cuts the x-axis at 6. Therefore, the equation of HK is \( x = 6 \).

(ii) **Required to find:** the set of 3 inequalities which define the shaded region in the diagram.

**Solution:**

Region shaded is on the left of \( x = 6 \). Hence, \( x \leq 6 \) (and including line).

The region shaded is on the side with the smaller angle. Hence, \( y \leq \frac{1}{6}x + 5 \) (including line).

The side shaded is that with the larger angle. Therefore, region is \( y \geq -\frac{5}{8}x + 5 \) (including line). Hence the three inequalities that define the shaded region are:

\[
\begin{align*}
x &\leq 6 \\
y &\leq \frac{1}{6}x + 5 \\
y &\geq -\frac{5}{8}x + 5
\end{align*}
\]
11. a. **Data:** P, Q are midpoints of \( \triangle XYZ \) with XP = 7.5 cm, XQ = 4.5 cm and area of \( \triangle XPQ = 13.5 \text{ cm}^2 \)

(i) **Required To Calculate:** the size of \( \angle PXQ \)

**Calculation:**

Let \( PXQ = \theta \)

\[ \therefore \frac{1}{2}(7.5)(4.5)\sin \theta = 13.5 \]

\[ \therefore \sin \theta = \frac{13.5 \times 2}{7.5 \times 4.5} \]

\[ = 0.8 \]

\[ \theta = 53.1^\circ \]

\[ = 53^\circ \text{ (to the nearest degree)} \]

(ii) **Required To Calculate:** Area of \( \triangle YXZ \)

**Calculation:**

\[ XY = 2(7.5) \]

\[ = 15 \]

\[ XZ = 2(4.5) \]

\[ = 9 \]

\[ \sin \theta = \frac{4}{5} \text{ or } 0.8 \]

\[ \therefore \text{ Area of } \triangle YXZ = \frac{1}{2}(15)(9) \times \frac{4}{5} \]

\[ = 54 \text{ square units} \]

**OR**
\[ \Delta XPQ \text{ and } \Delta XYZ \text{ are equivalent or similar.} \]
\[ XP : XY = 1 : 2 \]
\[ \therefore \text{Area of } \Delta XPQ : \text{Area of } \Delta XYZ = 1^2 : 2^2 \]
\[ = 1 : 4 \]
\[ \therefore \text{Area of } \Delta XYZ = 13.5 \times 4 \]
\[ = 54 \text{ cm}^2 \]

b. **Data:** Diagram of trapezium \( SJKM \) with \( SJ \) parallel to \( MK \), \( SM = SJ = 50 \text{ m} \), \( M\hat{J}K = 124^\circ \) and \( M\hat{S}T = 136^\circ \)

**Required To Calculate:**

(i) (a) \( \hat{SJM} \)
    
(b) \( \hat{JKM} \)

(ii) (a) \( MJ \)
    
(b) \( JK \)

**Calculation:**

(i) (a) \( \hat{SJM} = SMJ \) (base angles of isosceles triangle)

\[ \therefore \hat{SJM} = \frac{180^\circ - 136^\circ}{2} \]
\[ = 22^\circ \text{ (sum of angles in } \Delta = 180^\circ) \]

(b) \( \hat{JKM} = 22^\circ \) (alternate angles)

\[ \therefore \hat{JKM} = 180^\circ - (124^\circ + 22^\circ) \]
\[ = 34^\circ \text{ (sum of angles in } \Delta = 180^\circ) \]

(ii) (a) \[ \frac{MJ}{\sin 136^\circ} = \frac{50}{\sin 22^\circ} \text{ (sine rule)} \]

\[ \therefore MJ = \frac{50 \times \sin 136^\circ}{\sin 22^\circ} \]
\[ = 92.71 \text{ m} \]
\[ = 92.7 \text{ m to 1 decimal place} \]

**OR**
12. This question is not done since it involves latitude and longitude (Earth Geometry) which has been removed from the syllabus.

13. a. Data: $ABCD$ is a parallelogram with $\overline{DC} = 3x$, $\overline{DA} = 3y$ and $P$ on $DB$ such that $DP : PB = 1 : 2$.

(i) **Required To Express:** $\overline{AB}$ in terms of $x$ and $y$

**Solution:**

$\overline{AB} \equiv \overline{DC}$

(Equal in magnitude and parallel, as expected for opposite sides of a parallelogram).

$\therefore \overline{AB} = 3x$

(ii) **Required To Express:** $\overline{BD}$ in terms of $x$ and $y$

**Solution:**

Similarly as (a) $\overline{CB} \equiv \overline{DA} = 3y$

$\overline{BD} = \overline{BC} + \overline{CD}$

$= (3y) + (-3x)$

$= -3x - 3y$
(iii) **Required To Express:** $\overrightarrow{DP}$ in terms of $x$ and $y$

**Solution:**

\[
\overrightarrow{DB} = -(3x - 3y) = 3x + 3y
\]

Since $DP : PB = 1 : 2$, then $DP = \frac{1}{3} DB$ and

\[
\overrightarrow{DP} = \frac{1}{3} (3x + 3y) = x + y
\]

b. **Required To Prove:** $\overrightarrow{AP} = x - 2y$

**Proof:**

\[
\overrightarrow{AP} = \overrightarrow{AD} + \overrightarrow{DP} = -(3y) + (x + y) = x - 2y
\]

Q.E.D.

c. **Data:** $E$ is the midpoint of $DC$

**Required To Prove:** $A$, $P$ and $E$ are collinear.

**Solution:**

\[
\overrightarrow{DE} = \frac{1}{2} (3x) \\
\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE} = -(3y) + \frac{1}{2} x = \frac{1}{2} x - 3y
\]

\[
\frac{1}{2} x - 3y = \frac{1}{2} (x - 2y) = \frac{1}{2} \overrightarrow{AP}
\]
\( \overrightarrow{AE} \) is a scalar multiple of \( \overrightarrow{AP} \). Therefore, \( \overrightarrow{AE} \) is parallel to \( \overrightarrow{AP} \). Since A is a common point, P lies on AE and A, P and E are collinear.

d. **Data:** \( x = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \) and \( y = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \)

**Required To Prove:** \( \triangle AED \) is isosceles.

**Proof:**

\[
\overrightarrow{DA} = 3y = \begin{pmatrix} 3 \\ 0 \end{pmatrix}
\]

\[
\overrightarrow{DE} = \frac{1}{2}x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

\[
|\overrightarrow{DA}| = \sqrt{(3)^2 + (0)^2} = \sqrt{18}
\]

\[
|\overrightarrow{DE}| = \sqrt{(3)^2 + (0)^2} = 3
\]

\[
\overrightarrow{AE} = \frac{1}{2}x - 3y = \begin{pmatrix} 3 \\ -3 \end{pmatrix}
\]

\[
|\overrightarrow{AE}| = \sqrt{(0)^2 + (-3)^2} = 3
\]

In \( \triangle AED \) only 2 sides, \( AE \) and \( DE \), are equal, therefore the triangle is isosceles.

**Q.E.D.**
12. a. **Data:** \( M = \begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix} \)

(i) **Required To Prove:** \( M \) is a non-singular matrix

**Solution:**
\[
\text{Det } M = (2 \times 15) - (5 \times 7) = 30 - 35 = -5 \neq 0
\]
Hence, \( \exists M^{-1} \) and so \( M \) is non-singular.

(ii) **Required To Find:** \( M^{-1} \)

**Solution:**
\[
M^{-1} = -\frac{1}{5} \begin{pmatrix} 15 & -5 \\ -7 & 2 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 7 & -2 \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 7 & -2 \end{pmatrix}
\]

(iii) **Required To Find:** \( M \times M^{-1} \)

**Solution:**
\[
M \times M^{-1} = I \text{ where } I \text{ is the } 2 \times 2 \text{ identity matrix } = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},
\]
\[
M \cdot M^{-1} = \begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix} \cdot \begin{pmatrix} -3 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}
\]
\[
e_{11} = (2 \times -3) + \left(5 \times \frac{7}{5}\right) = -6 + 7 = 1
\]
\[ e_{12} = (2 \times 1) + \left(5 \times -\frac{2}{5}\right) \]
\[ = 2 - 2 \]
\[ = 0 \]

\[ e_{21} = (7 \times -3) + \left(15 \times \frac{7}{5}\right) \]
\[ = -21 + 21 \]
\[ = 0 \]

\[ e_{22} = (7 \times 1) + \left(15 \times -\frac{2}{5}\right) \]
\[ = 7 - 6 \]
\[ = 1 \]

\[ \therefore M \times M^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

\[ M \times M^{-1} = I \]

(iv) Required To Solve: \[
\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 17 \end{pmatrix}
\]

Solution:
\[
\begin{pmatrix} 2 & 5 \\ 7 & 15 \end{pmatrix} \times M^{-1} \begin{pmatrix} x \\ y \end{pmatrix} = M^{-1} \begin{pmatrix} -3 \\ 17 \end{pmatrix}
\]

\[
I \times \begin{pmatrix} x \\ y \end{pmatrix} = M^{-1} \begin{pmatrix} -3 \\ 17 \end{pmatrix}
\]

\[
\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 7 & -2 \end{pmatrix} \begin{pmatrix} -3 \\ 17 \end{pmatrix}
\]

\[
= \begin{pmatrix} (-3 \times -3) + (1 \times 17) \\ (7 \times -3) + (-2 \times 17) \end{pmatrix}
\]

\[
= \begin{pmatrix} 26 \\ -11 \end{pmatrix}
\]

Equating corresponding entries
\[ x = 26 \text{ and } y = -11 \]
b. (i) **Required To Find:** matrix $R$, which represents a reflection in the $y$– axis.

**Solution:**

The matrix, $R$, which represents a reflection in the $y$– axis is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

(ii) **Required To Find:** matrix $N$, which represents a clockwise rotation of $180^\circ$ about the origin.

**Solution:**

The matrix, $N$, which represents a clockwise rotation of $180^\circ$ about $O$ is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

(iii) **Required To Find:** matrix $T$, which represents a translation of $– 3$ units parallel to the $x$– axis and $5$ units parallel to the $y$– axis.

**Solution:**

A translation of $– 3$ units parallel to the $x$– axis (3 units horizontally to the left) and $5$ units parallel to the $y$– axis (5 units vertically upwards) may be represented by $T = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

(iv) **Data:**

$P \xrightarrow{RN} P'$, that is $N$ first, then $R$ second.

**Required To Find:** the coordinates of $P'$ and $P''$.

**Solution:**

\[
\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 11 \end{pmatrix} = \begin{pmatrix} (\(-1\times6\)) + (0\times11) \\ (0\times6) + (1\times11) \end{pmatrix} = \begin{pmatrix} 6 \\ -11 \end{pmatrix} \\
\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ -11 \end{pmatrix} = \begin{pmatrix} (\(-1\times6\)) + (0\times11) \\ (0\times6) + (1\times11) \end{pmatrix} = \begin{pmatrix} 6 \\ -11 \end{pmatrix}
\]

$\therefore P' = (6, -11)$.
\[ P \xrightarrow{NT} P^* \]
\[
\begin{pmatrix}
6 \\
11
\end{pmatrix}
\xrightarrow{\tau = \begin{pmatrix}
-3 \\
5
\end{pmatrix}}
\begin{pmatrix}
6 - 3 \\
11 + 5
\end{pmatrix}
= \begin{pmatrix}
3 \\
16
\end{pmatrix}
\]
\[
\begin{pmatrix}
3 \\
16
\end{pmatrix}
\xrightarrow{N}
\begin{pmatrix}
-1 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
3 \\
16
\end{pmatrix}
= \begin{pmatrix}
(-1 \times 3) + (0 \times 16) \\
(0 \times 3) + (-1 \times 16)
\end{pmatrix}
= \begin{pmatrix}
-3 \\
-16
\end{pmatrix}
\]
\[ \therefore P^* = (-3, -16) \]