Section I

1. a. (i) **Required To Calculate:** \((12.3)^2 - (0.246 \div 3)\) exactly.
   **Calculation:**
   \[
   (12.3)^2 - (0.246 \div 3) = 151.29 - 0.082 \\
   = 151.208 \text{ exactly}
   \]

   (ii) **Required To Calculate:** \((12.3)^2 - (0.246 \div 3)\) to 2 significant figures.
   **Calculation:**
   The number 151.208 = 150 to 2 significant figures.

b. **Data:** Table showing the depreciation of vehicles over a period.
   (i) **Required To Calculate:** The values of \(p\) and \(q\).
   **Calculation:**
   Taxi depreciates by 12% per year.
   
   .: Depreciation of taxi costing $40 000 after 1 year = \(\frac{12}{100} \times 40 000\)
   
   = $4800
   
   Hence, value after 1 year = $40 000\) - $4800
   
   = $35 200
   
   \(p = 35 200\)
   
   Depreciation of private car = $25 000\) - $21250
   
   = $3750
   
   % Depreciation = \(\frac{3750}{25 000} \times 100\)
   
   = 15%
   
   \(q = 15\)

   (ii) **Required To Calculate:** Value of taxi after 2 years.
   **Calculation:**
   Depreciation of taxi in the 2\(^{nd}\) year is 12% of its value after 1\(^{st}\) year.
   
   Depreciation in 2\(^{nd}\) year = \(\frac{12}{100} \times 35 200\)
   
   = $4 224
   
   .: Value of taxi after 2 years = $35 200\) - $4 224
   
   = $30 976

   **OR**
\[ A = P \left(1 - \frac{R}{100}\right)^n \]
\[ P = 40000 \quad R = -12 \quad n = 2 \]
\[ A = 40000 \left(1 - \frac{12}{100}\right)^2 \]
\[ = 30976 \]

c. **Data:** GUY $1.00 ≡ US$0.01 and EC $1.00 ≡ US$0.37

(i) **Required To Calculate:** Value of GUY $60 000 in US $.

**Calculation:**

GUY $1.00 ≡ US$0.01

GUY $60 000 = US$0.01 \times 60 000

= US$600.00

(ii) **Required To Calculate:** Value of US $925 in EC $.

**Calculation:**

US$0.37 ≡ US$1.00

US$1.00 = EC$ \frac{1.00}{0.37}

US$925.00 = EC$ \frac{1.00}{0.37} \times 925

= EC$2 500.00

2. a. **Required To Simplify:** \( \frac{x - 3}{3} - \frac{x - 2}{5} \)

**Solution:**

Simplifying

\[ \frac{x - 3}{3} - \frac{x - 2}{5} = \frac{5(x - 3) - 3(x - 2)}{15} \]
\[ = \frac{5x - 15 - 3x + 6}{15} \]
\[ = \frac{2x - 9}{15} \]
b. (i) Required To Factorise: (a) \( x^2 - 5x \), (b) \( x^2 - 81 \\

Factorising:
(a) \( x^2 - 5x = x \cdot x - 5 \cdot x \)  
\[ = x(x - 5) \]

(b) \( x^2 - 81 = (x)^2 - (9)^2 \)  
Difference of 2 squares.  
\[ = (x - 9)(x + 9) \]

(ii) Required To Simplify: \( \frac{a^2 + 4a}{a^2 + 3a - 4} \)

Solution:  
Simplifying  
\[ \frac{a^2 + 4a}{a^2 + 3a - 4} = \frac{a(a + 4)}{(a - 1)(a + 4)} \]
\[ = \frac{a}{a - 1} \]

c. Data: 2 cassettes and 3 CD’s cost $175 and 4 cassettes and 1 CD cost $125. One cassette costs $x and one CD costs $y.

(i) Required To Find: Expression in \( x \) and \( y \) for the information given.

Solution:
2 cassettes at $x each and 3 CD’s at $y each cost \( (2 \times x) + (3 \times y) \),  
Hence, \( 2x + 3y = 175 \) ...(1)  
4 cassettes and 1 CD cost \( (4 \times x) + (1 \times y) \),  
Hence, \( 4x + y = 125 \) ...(2)  

(ii) Required To Calculate: Cost of one cassette.

Calculation:
From (2)  
\[ y = 125 - 4x \]
Substitute in (1)  
\[ 2x + 3(125 - 4x) = 175 \]
\[ 2x + 375 - 12x = 175 \]
\[ 375 - 175 = 12x - 2x \]
\[ 10x = 200 \]
\[ x = 20 \]
\[ \therefore \text{Cost of one cassette is } \$20. \]
3. a. **Data:** Diagram of a quadrilateral $KLMN$ with $LM = LN = LK$, $\angle KLM = 140^\circ$ and $\angle LKN = 40^\circ$.

![Diagram of a quadrilateral KLMN with LM = LN = LK, angle KLM = 140° and angle LKN = 40°.](image)

(i) **Required To Calculate:** $\angle LNK$

**Calculation:**
- $LK = LN$ (data)
- $\angle LNK = 40^\circ$
- (Base angles of an isosceles triangle are equal).

(ii) **Required To Calculate:** $\angle NLM$

**Calculation:**
- $\angle NLM = 180^\circ - (40^\circ + 40^\circ)$
- $= 100^\circ$
- (Sum of angles in a triangle = 180°).
- $\angle NLM = 140^\circ - 100^\circ$
- $= 40^\circ$

(iii) **Required To Calculate:** $\angle KNM$

**Calculation:**
- $LN = LM$ (data)
- $\angle LNM = \angle LMN$
- $\angle LNM = \frac{180^\circ - 40^\circ}{2}$
- $= 70^\circ$
- (Base angles in an isosceles triangle are equal and sum of angles in a triangle = 180°).
- $\angle KNM = 40^\circ + 70^\circ$
- $= 110^\circ$
b. **Data:** Survey done on 39 students on the ability to ride a bike and/or drive a car.

   (i) **Required To Complete:** Venn diagram to represent the information given.
   **Solution:**
   
   ![Venn Diagram](image)

   (ii) **Required To Find:** Expression in $x$ for the number of students in the survey.
   **Solution:**
   No. of students in the survey $= (18 - x) + x + (15 - x) + 3x$
   $= 33 + 2x$

   (iii) **Required To Calculate:** $x$
   **Calculation:**
   Hence,
   $33 + 2x = 39$
   $2x = 39 - 33$
   $x = 3$

4. **Data:** $AB = 8$ cm, $\hat{BAC} = 60^\circ$ and $AC = 5$ cm

   a. **Required To Construct:** Triangle $ABC$ based on the information given.
   **Solution:**
   
   ![Triangle ABC](image)
b. **Required To Find:** Length of $BC$
   **Solution:**
   $BC = 7$ cm (by measurement)

c. **Required To Calculate:** Perimeter of $\triangle ABC$
   **Calculation:**
   Perimeter of $\triangle ABC = 5$ cm + 8 cm + 7 cm  
   $= 20$ cm

d. **Required To Draw:** Line $CD$ which is perpendicular to $AB$ and meets $AB$ at $D$.
   **Solution:**

![Diagram of triangle ABC with perpendicular line CD]

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e. **Required To Find:** The length of $CD$.
   **Solution:**
   $CD = 4.3$ cm (by measurement)

f. **Required To Calculate:** Area of $\triangle ABC$
   **Calculation:**
   Area of $\triangle ABC = \frac{8 \times 4.3}{2} = 17.2$ cm$^2$
5. **Data:** Diagram illustrating the graph of the function \( f(x) = x^2 - 2x - 3 \) for \( a \leq x \leq b \) and the tangent at (2, -3).

   a. **Required To Find:** \( a \) and \( b \).

   **Solution:**

\[
\begin{align*}
x & \geq -2 \quad \text{and} \quad x \leq 4. \\
\therefore \quad a & = -2 \quad \text{and} \quad b = 4 \quad \text{from the diagram, that is} \quad -2 \leq x \leq 4.
\end{align*}
\]

b. **Required To Find:** \( x \) for \( x^2 - 2x - 3 = 0 \).

   **Solution:**

\[
\begin{align*}
& \quad \text{cuts the } x \text{ – axis at } -1 \text{ and } 3 \text{ as seen on the diagram. Therefore,} \\
& \quad \text{the values of } x \text{ are } -1 \text{ and } 3.
\end{align*}
\]
c. **Required To Find:** Coordinates of the minimum point on the graph.

**Solution:**

The minimum point of $f(x)$ is $(1, -4)$ as seen on the diagram.

d. **Required To Find:** Whole number values of $x$ for which $x^2 - 2x - 3 < 1$.

**Solution:**

From the diagram, $x^2 - 2x - 3 < 1$ for $x > -1.2$ and $x < 3.2$, that is $-1.2 < x < 3.2$.

$x \in W \quad \therefore x = \{0, 1, 2, 3\}$
e. **Required To Find:** gradient of \( f(x) = x^2 - 2x - 3 \) at \( x = 2 \).

**Solution:**

Choosing \((2, -3)\) and \((4, 1)\) as 2 points on the tangent to \( f(x) \) at \((2, -3)\).

Gradient

\[
\begin{align*}
\text{Gradient} &= \frac{1 - (-3)}{4 - 2} \\
&= \frac{4}{2} \\
&= 2
\end{align*}
\]

\(\therefore\) Gradient of \( f(x) \) at \((2, -3)\) is 2.

6. **Data:** Diagram showing the direction and distance of a man walking.

a. **Required To Complete:** The diagram given showing distances \( x \) km, \((x + 7)\) km, and 13 km.

**Solution:**

![Diagram showing the direction and distance of a man walking.](image)
b. **Required To Find:** Equation in \( x \) that satisfies Pythagoras’ Theorem and that simplifies to \( x^2 + 7x - 60 = 0 \).

**Solution:**

\[
(x)^2 + (x + 7)^2 = (13)^2 \quad \text{(Pythagoras’ Theorem)}
\]

\[
x^2 + (x^2 + 14x + 49) = 168
\]

\[
2x^2 + 14x - 120 = 0
\]

\[
\div 2
\]

\[
x^2 + 7x - 60 = 0
\]

Q.E.D.

c. **Required To Find:** Distance \( GH \).

**Solution:**

\[
x^2 + 7x - 60 = 0
\]

\[
(x + 12)(x - 5) = 0
\]

\[
x = -12 \text{ or } 5
\]

\[
x \neq -12 \text{ (since } GH \text{ and } HF \text{ would be negative)}
\]

\[
x = 5 \text{ only}
\]

\[
GH = 5 \text{ km}
\]

d. **Required To Find:** Bearing of \( F \) from \( G \).

**Solution:**

The bearing of \( F \) from \( G \) is illustrated by \( \theta \).

\[
\tan \theta = \frac{12}{5}
\]

\[
\theta = \tan^{-1}\left(\frac{12}{5}\right)
\]

\[
\theta = 67.4^\circ
\]

\[
\therefore \text{ The bearing of } F \text{ from } G \text{ is } 067.4^\circ
\]
7. **Data:** Table showing the gains in mass of 100 cows over a certain period.
   a. **Required To Complete:** Table of information given.
      
      **Solution:**
      Modifying the table for the data of the continuous variable

<table>
<thead>
<tr>
<th>Gain in mass in kg</th>
<th>L.C.B</th>
<th>U.C.B.</th>
<th>Mid-class Interval, ( \frac{L.C.B. + U.C.B.}{2} )</th>
<th>Frequency, ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 – 9</td>
<td>2</td>
<td>2</td>
<td>( \frac{4.5 + 9.5}{2} = 7 )</td>
<td>2</td>
</tr>
<tr>
<td>10 – 14</td>
<td>29</td>
<td>2</td>
<td>( \frac{9.5 + 14.5}{2} = 12 )</td>
<td>29</td>
</tr>
<tr>
<td>15 – 19</td>
<td>37</td>
<td>2</td>
<td>( \frac{14.5 + 19.5}{2} = 17 )</td>
<td>37</td>
</tr>
<tr>
<td>20 – 24</td>
<td>37</td>
<td>2</td>
<td>( \frac{19.5 + 24.5}{2} = 22 )</td>
<td>16</td>
</tr>
<tr>
<td>25 – 29</td>
<td>37</td>
<td>2</td>
<td>( \frac{24.5 + 29.5}{2} = 27 )</td>
<td>14</td>
</tr>
<tr>
<td>30 – 34</td>
<td>37</td>
<td>2</td>
<td>( \frac{29.5 + 34.5}{2} = 32 )</td>
<td>2</td>
</tr>
</tbody>
</table>

b. (i) **Required To Estimate:** Mean gain in mass of the 100 cows.
   
   **Solution:**
   The mean gain, \( \bar{x} \)
   \[
   \bar{x} = \frac{\sum fx}{\sum f} = \frac{(2 \times 7) + (29 \times 12) + (37 \times 17) + (16 \times 22) + (14 \times 27) + (2 \times 32)}{\sum f = 100}
   \]
   \[
   = \frac{17.85 \text{ kg}}{}
   \]

   (ii) **Required To Draw:** The frequency polygon for the information given.
   
   **Solution:**
   The points (2, 0) and (37, 0) are obtained by extrapolation as the frequency polygon is to be bounded by the horizontal axis.
c. **Required To Calculate:** Probability that a randomly chosen cow gained 20 kg or more.

**Solution:**

\[
P(\text{cow gained } \geq 20 \text{ kg}) = \frac{\text{No. of cows gaining } \geq 20 \text{ kg}}{\text{Total no. of cows}}
\]

\[
= \frac{16 + 14 + 2}{\sum f = 100}
\]

\[
= \frac{32}{100}
\]

\[
= \frac{8}{25}
\]
8. **Data:** Drawings showing a sequence of squares made from toothpicks.
   a. (i) **Required To Draw:** Next shape in the sequence.
      
      ![Drawing of squares made from toothpicks]
      
      **Solution:**

   (ii) The column 2 is a product of three numbers, that is

   \[
   (n + 1) \times 2 \times n \times 2.
   \]

   a) **Required To Complete:** Table when \( n = 4 \)

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( n ), of one side of square</td>
<td>Pattern for calculating number of toothpicks in square</td>
<td>Total number of toothpicks in square</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>( 1 \times 2 \times 2 )</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 3 \times 2 )</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 4 \times 2 )</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 5 \times 2 )</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>( 7 \times 8 \times 2 )</td>
<td>112</td>
</tr>
</tbody>
</table>

   \[
   n = 10 \rightarrow 10 \times 11 \times 2 = 220
   \]

   \[
   r = n \times (n + 1) \times 2
   \]

   \[
   s = 10 \rightarrow r = 2(10 + 1) \times 2 = 220
   \]

   \[
   2n(n + 1)
   \]

   \[
   2n(10 + 1)
   \]
Solution:
When column 1 is 4

Column 2 = \(4 \times (4 + 1) \times 2\)
\[= 4 \times 5 \times 2\]

Column 3 is the result = 40 of column 2.

b) **Required To Complete:** The table when \(n = 7\)

Solution:
When column 1 is 7

Column 2 = \(7 \times (7 + 1) \times 2\)
\[= 7 \times 8 \times 2\]

And column 3 is 112.

b. (i) **Required To Complete:** The table for length of side \(n\).

Solution:
When column 1 is \(n\), column 2 is \(r\).
\[\therefore r = n \times (n + 1) \times 2\]
\[= 2n(n + 1)\]

Col 3 = \(2n(n + 1)\)

(ii) **Required To Complete:** The table when column 3 is 220.

Solution:
Column 3 is 220.

\[n \times (n + 1) \times 2 = 220\]
\[2n(n + 1) = 220\]
\[n(n + 1) = 110\]
\[n^2 + n - 110 = 0\]
\[(n + 11)(n - 10) = 0\]
\[n = -11 \text{ or } 10\]
\[ n \neq -ve \]
\[ n = 10 \]

Therefore, in (b) (ii) \( s = 10 \) and

Column 2 \( = 10 \times (10 + 1) \times 2 \)
\( = 10 \times 11 \times 2 \)

9. a. **Data:** \( y = x + 2 \) and \( y = x^2 \)

**Required To Calculate:** \( x \) and \( y \)

**Calculation:**

Let \( y = x + 2 \ldots (1) \) and \( y = x^2 \ldots (2) \)

Equating

\[ x^2 = x + 2 \]
\[ x^2 - x - 2 = 0 \]
\[ (x - 2)(x + 1) = 0 \]

\[ \therefore x = 2 \text{ or } -1 \]

When \( x = 2 \)

\[ y = 2 + 2 = 4 \]

When \( x = -1 \)

\[ y = (-1)^2 = 1 \]

Hence, \( x = 2 \) and \( y = 4 \) **OR** \( x = -1 \) and \( y = 1 \).

b. **Data:** Strip of wire 32 m long is cut into 2 pieces and formed into a square and a rectangle.

\[ \begin{array}{c}
\text{Square} \\
x \text{ cm} \\
\hline
\text{Rectangle} \\
l \text{ cm} \\
3 \text{ cm}
\end{array} \]

(i) **Required To Find:** Expression in terms of \( x \) and \( l \) for the length of the strip of wire.

**Solution:**

Perimeter of square \( = (x \times 4) \)
\( = 4x \text{ cm} \)

Perimeter of rectangle \( = 2(l + 3) \)
\( = 2l + 6 \text{ cm} \)
\[ 4x + 2l + 6 = 32 \]

(ii) **Required To Prove:** \( l = 13 - 2x \)

**Proof:**
\[
\begin{align*}
4x + 2l + 6 &= 32 \\
4x + 2l &= 32 - 6 \\
4x + 2l &= 26 \\
\div 2 \\
2x + l &= 13 \\
l &= 13 - 2x
\end{align*}
\]

(iii) **Required To Prove:** \( S = x^2 - 6x + 39 \).

**Proof:**
\[
S = (x^2) + (3)(l)
\]
\[
S = x^2 + 3l
\]
\[
S = x^2 + 3(13 - 2x)
\]
\[
= x^2 + 39 - 6x
\]
\[
= x^2 - 6x + 39
\]

Q.E.D.

(iv) **Required To Calculate:** \( x \) for which \( S = 30.25 \)

**Calculation:**
\[
x^2 - 6x + 39 = 30.25
\]
\[
x^2 - 6x + 8.75 = 0
\]
\[
\times 4
\]
\[
4x^2 - 24x + 35 = 0
\]
\[
(2x - 5)(2x - 7) = 0
\]
\[
x = \frac{2}{2} \text{ or } 3 \frac{1}{2}
\]

Hence, when \( S = 30.25 \), \( x = \frac{2}{2} \) or \( 3 \frac{1}{2} \).

10. **Data:** Conditions for the parking of \( x \) vans and \( y \) cars at a lot.

(i) **Required To Find:** Inequality for the information given.

**Solution:**
No. of vans = \( x \)
No. of cars = \( y \)
Lot has space for no more than 60 vehicles. Therefore,
\[ x + y \leq 60 \quad ...(1) \]
(ii) **Data:** Owner must part at least 10 cars.
**Required To Find:** Inequality for the information given.
**Solution:**
No. of cars is at least 10.
\[ y \geq 10 \] \hspace{1cm} (2)

(iii) **Data:** Number of cars parked must be fewer than or equal to twice the number of vans parked.
**Required To Find:** Inequality for the information given.
**Solution:**
The no. of cars parked must be fewer than or equal to twice the number of vans.
\[ y \leq 2x \] \hspace{1cm} (3)

(iv) **Required To Draw:** The graphs of the lines associated with the inequalities and shaded the region which satisfies all three.
**Solution:**
Obtaining 2 points on the line \( x + y = 60 \).

When \( x = 0 \) \hspace{1cm} 0 + y = 60
\[ y = 60 \]

The line \( x + y = 60 \) passes through the point (0, 60).

When \( y = 0 \) \hspace{1cm} x + 0 = 60
\[ x = 60 \]

The line \( x + y = 60 \) passes through the point (60, 0).

The side with the smaller angle satisfies the \( \leq \) region.
The region which satisfies \( x + y \leq 60 \) is
The line $y = 10$ is a horizontal straight line.
The region which satisfies $y \geq 10$ is

Obtaining 2 points on the line $y = 2x$.
The line $y = 2x$ passes through the origin $(0, 0)$.
When $x = 20 \quad y = 2(20) \quad y = 40$
The line $y = 2x$ passes through the point $(20, 40)$.

The side with the smaller angle satisfies the $\leq$ region.
The region which satisfies $y \leq 2x$ is

The region which satisfies all three inequalities is the area in which all three shaded regions overlap.
Data: Parking fee for a van is $6 and parking fee for a car is $5.
**Required To Find:** Expression in $x$ and $y$ for total fees charged for parking $x$ vans and $y$ cars.

**Solution:**
The total fees on $x$ vans at $6 \text{ each}$ and $y$ cars at $5 \text{ each}$

$$= (x \times 6) + (y \times 5)$$

$$= 6x + 5y$$

(vi) **Required To Find:** Vertices of the shaded region.

**Solution:**
The vertices are $(5, 10)$, $(20, 40)$ and $(50, 10)$.

(vii) Required To Calculate: Maximum fees charged.

**Calculation:**
Testing $(20, 40)$ and $(50, 10)$

$x = 20$ \hspace{1cm} $y = 40$

Fees $= 6(20) + 5(40)$

$= 320$

$x = 50$ \hspace{1cm} $y = 10$

Fees $= 6(50) + 5(10)$

$= 350$

$\therefore$ Maximum fee charged is $350$, when there are 50 vans and 10 cars.

11. a. **Data:** Diagram of a vertical tower and antenna mounted atop. Point P lies on horizontal ground.

(i) **Required To Complete:** The diagram given, showing the distance 28 m, angles $40^\circ$ and $54^\circ$ and any right angles.

**Solution:**

(ii) **Required To Calculate:** Length of antenna $TW$.

**Calculation:**
\[ \frac{TF}{28} = \tan 40^\circ \]

\[ TF = 28 \tan 40^\circ \]

\[ \frac{WF}{28} = \tan 54^\circ \]

\[ WF = 28 \tan 54^\circ \]

Length of antenna = Length of \( WF \) – Length of \( TF \)

\[ = 28 \tan 54^\circ - 28 \tan 40^\circ \]

\[ = 15.04 \text{ m} \]

\[ = 15.0 \text{ m} \]

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b. **Data:** Diagram showing a circle centre \( O \) and tangents \( BD \) and \( DCE \). \( B \hat{C}D = 70^\circ \)

![Diagram of a circle with tangents and angles](image)

(i) **Required To Calculate:** \( O \hat{C}E \)

**Calculation:**

\( O \hat{C}E = 90^\circ \)

(Angles made by tangent to a circle and radius, at point of contact = 90°).

(ii) **Required To Calculate:** \( \hat{B}AC \)

**Calculation:**

\[ \hat{B}AC = \frac{1}{2} (140^\circ) \]

\[ = 70^\circ \]
(Angles subtended by a chord at the centre of the circle equal twice the angle is subtends at the circumference, standing on the same arc).

(iii) **Required To Calculate:** $B\hat{O}C$
**Calculation:**
\[O\hat{C}B = 180^\circ - (70^\circ + 90^\circ)\]
\[= 20^\circ\]
(Angles in a straight line).
\[OB = OC \quad \text{(radii)}\]
\[\hat{O}B\hat{C} = 20^\circ\]
(Base angles of an isosceles triangle are equal).
\[B\hat{O}C = 180^\circ - (20^\circ + 20^\circ)\]
\[= 140^\circ\]
(Sum of angles in a triangle = 180°).

(iv) **Required To Calculate:** $B\hat{D}C$
**Calculation:**
\[B\hat{D}C = 360^\circ - (90^\circ + 90^\circ + 140^\circ)\]
\[= 40^\circ\]
(Sum of angles in a quadrilateral is 360°).

12. a. **Data:** Parallelogram $EFGH$ with $EH = 4.2$ cm, $EF = 6$ cm and $\hat{H}\hat{E}F = 70^\circ$

(i) **Required To Calculate:** Length of $HF$.
**Calculation:**
\[HF^2 = (4.2)^2 + (6)^2 - 2(4.2)(6)\cos 70^\circ \quad \text{(Cosine Rule)}\]
\[= 36.402\]
\[HF = \sqrt{36.402}\]
\[= 6.033\]
\[= 6.03 \text{ to 2 decimal places}\]
(ii) **Required To Calculate:** Area of parallelogram $EFGH$.

**Calculation:**

Area of $\triangle HEF = \frac{1}{2} (4.2)(6) \sin 70^\circ$

Diagonal $HF$ bisects the parallelogram $EFGH$.

$\therefore$ Area of parallelogram $EFGH = 2 \left( \frac{1}{2} (4.2)(6) \sin 70^\circ \right)$

$= 23.680$

$= 23.68$ to 2 decimal places

b. This part of the question has not been solved as it involves Earth Geometry which has since been removed from the syllabus.

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13. **Data:** Diagram showing the position vectors of 2 points $A$ and $C$ relative to $O$.

a. **Required To Complete:** The diagram to show $B$, such that $OABC$ is a parallelogram and $\overrightarrow{u}$.

**Solution:**

![Diagram showing position vectors of points A, B, C, and G]

$\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ from diagram
\[ u = \overrightarrow{OA} + \overrightarrow{OC} \]
\[ = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} \]
\[ = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \]

b. (i) **Required To Express**: \( \overrightarrow{OA} \) in the form \( \begin{pmatrix} x \\ y \end{pmatrix} \).

**Solution:**
Since \( A \) is \((6, 2)\) then \( \overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} \) is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = 6 \) and \( y = 2 \).

(iii) **Required To Express**: \( \overrightarrow{OC} \) in the form \( \begin{pmatrix} x \\ y \end{pmatrix} \).

**Solution:**
Since \( C \) is \((0, 4)\) then \( \overrightarrow{OC} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \) is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = 0 \) and \( y = 4 \).

(iv) **Required To Express**: \( \overrightarrow{AC} \) in the form \( \begin{pmatrix} x \\ y \end{pmatrix} \).

**Solution:**
\[ \overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} \]
\[ = -\begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} \]
\[ = \begin{pmatrix} -6 \\ 2 \end{pmatrix} \]
\[ \overrightarrow{AC} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} \] is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = -6 \) and \( y = 2 \).

c. **Data**: \( G \) is the midpoint of \( OB \).

(i) **Required To Find**: Coordinates of \( G \).

**Solution:**
\[ \overrightarrow{OB} = \begin{pmatrix} 6 \\ 6 \end{pmatrix} \]
\[ \overrightarrow{OG} = \frac{1}{2} \overrightarrow{OB} \]
\[ = \frac{1}{2} \begin{pmatrix} 6 \\ 6 \end{pmatrix} \]
\[ = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \]
Hence G is (3, 3)

(ii) Required To Prove: A, G and C lie on a straight line.
Proof:
\[ \overrightarrow{AC} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} \]
\[ \overrightarrow{AG} = \overrightarrow{AO} + \overrightarrow{OG} \]
\[ = \begin{pmatrix} 6 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} \]
\[ = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \]
\[ = \frac{1}{2} \overrightarrow{AC} \]
\[ \overrightarrow{AG} \] is a scalar multiple of \( \overrightarrow{AC} \). \( \overrightarrow{AG} \) and \( \overrightarrow{AC} \) are parallel. G is a common point, therefore, G lies on \( \overrightarrow{AC} \), hence, A, G and C lies on the same straight line, that they are collinear.

14. a. Data: \[ |M| = \begin{pmatrix} 2 & 3 \\ -1 & x \end{pmatrix} = 9 \]
(i) Required To Calculate: \( a \)
Calculation:
\[ |M| = 9 \]
\[ (2 \times x) - (3 \times -1) = 9 \]
\[ 2x + 3 = 9 \]
\[ 2x = 6 \]
\[ x = 3 \]

(ii) Required To Calculate: \( M^{-1} \)
Calculation:
\[
\begin{align*}
M &= \begin{pmatrix} 2 & 3 \\ -1 & 3 \end{pmatrix} \\
M^{-1} &= \frac{1}{9} \begin{pmatrix} 3 & -3 \\ -(-1) & 2 \end{pmatrix} \\
&= \begin{pmatrix} \frac{3}{9} & -\frac{3}{9} \\ \frac{1}{9} & \frac{2}{9} \end{pmatrix}
\end{align*}
\]

(iii) Required To Prove: \( M^{-1}M = I \)
Proof:
\[
M_{2 \times 2} \times M_{2 \times 2}^{-1} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}
\]
\[
e_{11} = \left( 2 \times \frac{3}{9} \right) + \left( 3 \times \frac{1}{9} \right) \\
= \frac{9}{9} \\
= 1
\]
\[
e_{12} = \left( 2 \times -\frac{3}{9} \right) + \left( 3 \times \frac{2}{9} \right) \\
= \frac{0}{9} \\
= 0
\]
\[
e_{21} = \left( -1 \times \frac{3}{9} \right) + \left( 3 \times \frac{1}{9} \right) \\
= \frac{0}{9} \\
= 0
\]
\[
e_{22} = \left( -1 \times -\frac{3}{9} \right) + \left( 3 \times \frac{2}{9} \right) \\
= \frac{9}{9} \\
= 1
\]
\[
M \times M^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
= I
\]

Q.E.D.
b. **Data:** Graph showing line segment $AC$ and its image $A'C'$ after a transformation $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$.

![Graph showing line segment AC and its image A'C' after a transformation](image)

(i)  
(a) **Required To Express:** $A$ and $C$ as a single $2 \times 2$ matrix.  
**Solution:**  
Coordinates of $A$ and $C$ in matrix form is $\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix}$.

(b) **Required To Express:** $A'$ and $C'$ as a single $2 \times 2$ matrix.  
**Solution:**  
Coordinates of $A'$ and $C'$ in matrix form is $\begin{pmatrix} 2 \\ -4 \\ -3 \end{pmatrix}$.

(ii) **Required To Find:** Equation to represent the transformation of $AC$ onto $A'C'$.  
**Solution:**  
$$AC \rightarrow A'C'$$  
$$\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ -4 & -3 \end{pmatrix}$$

(iii) **Required To Calculate:** $p, q, r$ and $s$  
**Calculation:**  
Equating corresponding entries
Similarly,
\[ 10p + 20q = 10 \]
\[ -10p - 6q = -10 \]
\[ 14q = 0 \]
\[ \therefore q = 0 \text{ and } p = 1 \]
Similarly,
\[ 5r + 3s = -3 \quad \ldots (4) \]
\[ -10r - 6s = 6 \]
\[ 10r + 20s = -20 \]
\[ -10r - 6s = 6 \]
\[ 14s = -14 \]
\[ s = -1 \text{ and } r = 0 \]
\[ \therefore p = 1, q = 0, r = 0 \text{ and } s = -1 \text{ and the matrix } \begin{pmatrix} p & q \\ r & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \]
which represents a reflection in the \( x \)-axis.
We may also deduce this by observing the object \( AC \) and its image \( A'C' \).