JANUARY 2007 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) **Required To Calculate:** \(5.24(4 - 1.67)\)

   **Calculation:**
   
   \[
   5.24(4 - 1.67) = 5.24(2.33) \\
   = 12.2092 \quad \text{(exactly)} \\
   = 12.2 \text{ to 1 decimal place}
   \]

(ii) **Required To Calculate:** \(\frac{1.68}{1.5^2 - 1.45}\)

   **Calculation:**
   
   \[
   \frac{1.68}{1.5^2 - 1.45} = \frac{1.68}{2.25 - 1.45} \\
   = \frac{1.68}{0.8} \\
   = 2.1 \text{ (exactly)}
   \]

b. **Data:** Aaron received 2 shares totaling $60 from a sum shared in the ratio 2 : 5.

   **Required To Calculate:** The sum of money.

   **Calculation:**
   
   Aaron’s 2 shares total $60
   
   \[\therefore \text{1 share} = \frac{60}{2} = 30\]
   
   Total no. of shares = 2 + 5 = 7
   
   Sum that was shared altogether
   
   \[= 30 \times 7 = 210\]

   **Data:** Cost of gasoline is $10.40 for 3 litres. All currency in $EC.

   (i) **Required To Calculate:** Cost of 5 litres of gasoline.

   **Calculation:**
   
   If 3 litres of gasoline cost $10.40
   
   Then 1 litre of gasoline costs \(\frac{10.40}{3}\)
   
   And 5 litres of gasoline cost \(\frac{10.40}{3} \times 5\)
   
   \[= \frac{17.333}{3} = 17.33 \text{ to nearest cent}\]
(ii) **Required To Calculate:** Volume of gasoline that can be bought with $50.00.  
**Calculation:**  
$10.40$ affords $3$ litres  
$1.00$ will afford \( \frac{3}{10.40} \) litres  
$50.00$ will afford \( \left( \frac{3}{10.40} \times 50.00 \right) \) litres  
\[ = 14.4 \text{ litres} \]  
\[ = 14 \text{ litres to the nearest whole number} \]

2. a. **Data:** \( a = 2, \ b = -3 \) and \( c = 4 \)  
   (i) **Required To Calculate:** \( ab - bc \)  
   **Calculation:**  
   \[ ab - bc = 2(-3) - (-3)4 \]  
   \[ = -6 + 12 \]  
   \[ = 6 \]  
   (ii) **Required To Calculate:** \( b(a - c)^2 \)  
   **Calculation:**  
   \[ b(a - c)^2 = -3(2 - 4)^2 \]  
   \[ = -3(-2)^2 \]  
   \[ = -3(4) \]  
   \[ = -12 \]  

b. (i) **Data:** \( \frac{x}{2} + \frac{x}{3} = 5 \)  
   **Required To Find:** \( x \) where \( x \in Z \)  
   **Solution:**  
   \[ \frac{x}{2} + \frac{x}{3} = \frac{5}{1} \]  
   \[ \times 6 \]  
   \[ 6\left(\frac{x}{2}\right) + 6\left(\frac{x}{3}\right) = 6\left(\frac{5}{1}\right) \]  
   \[ 3x + 2x = 30 \]  
   \[ 5x = 30 \]  
   \[ x = 6 \in Z \]  
   **OR**
\[
\frac{x}{2} + \frac{x}{3} = 5
\]
\[
\frac{3(x) + 2(x)}{6} = 5
\]
\[
\frac{5x}{6} = 5
\]
\[
\times 6
\]
\[
5x = 30
\]
\[
x = 6 \in \mathbb{Z}
\]

(ii) **Data:** \(4 - x \leq 13\)

**Required To Find:** \(x\) where \(x \in \mathbb{Z}\)

**Solution:**
\[
4 - x \leq 13
\]
\[
-x \leq 13 - 4
\]
\[
\times -1
\]
\[
x \geq -9
\]

That is \(x = \{-9, -8, -7, \ldots, x \in \mathbb{Z}\}\)

c. **Data:** 1 muffin costs \(m\) and 3 cupcakes cost \(2m\)

(i) (a) **Required To Find:** Cost of five muffins in terms of \(m\).

**Solution:**
1 muffin costs \(m\)
5 muffins costs \(\$(m \times 5)\)
\[
= 5m
\]

(b) **Required To Find:** Cost of six cupcakes in terms of \(m\).

**Solution:**
If 3 cupcakes cost \(2m\)
Then 1 cupcake costs \(\frac{2m}{3}\)
And 6 cupcakes cost \(\frac{2m}{3} \times 6\)
\[
= 4m
\]

(ii) **Required To Find:** An equation for the total cost of 5 muffins and 6 cupcakes is \(\$31.50\).

**Solution:**
\[
5m + 4m = 9m
\]
Hence, \(9m = 31.50\)
\[
9m = 31.50
\]
3. a. (i) **Data:**

![Venn Diagram](image)

**Required To Describe:** The shaded region using set notation.

**Solution:**

The region shaded is all of sets $A$ and $B$, that is $A \cup B$.

(ii) **Data:**

![Venn Diagram](image)

**Required To Describe:** The shaded region using set notation.

**Solution:**

The region shaded is the region in $U$ except $A \cup B$, that is $(A \cup B)'$.

(iii) **Data:**

![Venn Diagram](image)

**Required To Describe:** The shaded region using set notation.

**Solution:**

The region shaded in the set $A$ only. Hence the shaded region is $A$.

b. **Data:** $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$P = \{\text{Prime numbers}\}$

$Q = \{\text{Odd numbers}\}$

**Required To Draw:** A Venn diagram to represent the information given.

**Solution:**

$P = \{2, 3, 5, 7\}$

$Q = \{1, 3, 5, 7, 9\}$
c. **Data:** Venn diagram illustrating the number of elements in each region.

(i) **Required To Find:** No. of elements in \( A \cup B \).

**Solution:**

\[
A \cup B = 10 + 4 + 3 = 17
\]

(ii) **Required To Find:** No. of elements in \( A \cap B \).

**Solution:**

\[
A \cap B = 4
\]

(iii) **Required To Find:** No of elements in \( (A \cap B)' \).

**Solution:**

\[
(A \cap B)' = 10 + 3 + 8 = 21
\]
(iv) **Required To Find:** No. of elements in $U$.

**Solution:**

![Image of set $U$ with elements 10, 4, 3, 8]

$$n(U) = 10 + 4 + 3 + 8$$

$$= 25$$

4. a. (i) **Required To Construct:** $\triangle ABC$ with $BC = 6\text{cm}$ and $AB = AC = 8\text{cm}$.

**Solution:**

![Diagram of triangle $ABC$ with side lengths]

(ii) **Required To Construct:** $AD$ such that $AD$ meets $BC$ at $D$ and is perpendicular to $BC$. 
(iii) (a) **Required To Find:** Length of $AD$.
**Solution:**
$AD = 7.3$ cm (by measurement)

(b) **Required To Find:** Size of $\hat{ABC}$
**Solution:**
$\hat{ABC} = 68^\circ$ (by measurement)
b. **Data:** \( P = (2, 4) \) and \( Q = (6, 10) \)

(i) **Required To Calculate:** Gradient of \( PQ \).
**Calculation:**
Gradient of \( PQ \) = \( \frac{10 - 4}{6 - 2} \)
= \( \frac{6}{4} \)
= \( \frac{3}{2} \)

(ii) **Required To Calculate:** Midpoint of \( PQ \).
**Calculation:**
Let midpoint of \( PQ \) be \( M \).
\[
M = \left( \frac{2 + 6}{2}, \frac{4 + 10}{2} \right)
\]
= \( (4, 7) \)

5. a. **Data:** \( f(x) \to 7x + 4 \) and \( g(x) \to \frac{1}{2x} \)

(i) **Required To Calculate:** \( g(3) \)
**Calculation:**
\[
g(3) = \frac{1}{2(3)}
\]
= \( \frac{1}{6} \)

(ii) **Required To Calculate:** \( f(-2) \)
**Calculation:**
\[
f(-2) = 7(-2) + 4
\]
= \( -14 + 4 \)
= \( -10 \)
(iii) Required To Calculate: \( f^{-1}(11) \)

Calculation:
Let \( y = 7x + 4 \)
\[ y - 4 = 7x \]
\[ \frac{y - 4}{7} = x \]
Replace \( y \) by \( x \)
\[ \therefore f^{-1}(x) = \frac{x - 4}{7} \]
\[ \therefore f^{-1}(11) = \frac{11 - 4}{7} \]
\[ = \frac{7}{7} \]
\[ = 1 \]

b. (i) \( x = 5 \)
\[ A'' = (1,2) \]
(ii) \( B'' = (3,2) \)
\[ C'' = (3,-1) \]
(iii) Reflection in the line \( y = 4 \)

6. Data: Table showing a frequency distribution of scores of 100 students in an examination.
(i) Required To Complete: And modify the table given.
Solution:

<table>
<thead>
<tr>
<th>Score (Discrete Variable)</th>
<th>U.C.B</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
<th>Points to Plot (U.C.B, C.F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 – 25</td>
<td>25</td>
<td>5</td>
<td>5</td>
<td>(25, 0)</td>
</tr>
<tr>
<td>26 – 30</td>
<td>30</td>
<td>18</td>
<td>18 + 5 = 23</td>
<td>(30, 23)</td>
</tr>
<tr>
<td>31 – 35</td>
<td>35</td>
<td>23</td>
<td>23 + 23 = 46</td>
<td>(35, 46)</td>
</tr>
<tr>
<td>36 – 40</td>
<td>40</td>
<td>22</td>
<td>22 + 46 = 68</td>
<td>(40, 68)</td>
</tr>
<tr>
<td>41 – 45</td>
<td>45</td>
<td>21</td>
<td>21 + 68 = 89</td>
<td>(45, 89)</td>
</tr>
<tr>
<td>46 – 50</td>
<td>50</td>
<td>11</td>
<td>11 + 89 = 100</td>
<td>(50, 100)</td>
</tr>
</tbody>
</table>

\[ \sum f = 100 \]

The point (20, 0) corresponding to an upper class boundary of 20 and a cumulative frequency value of 0, obtained by checking ‘backwards’, is to be plotted, as the graph of cumulative frequency starts from the horizontal axis.
(ii) **Data:** Scale is 2 cm to represent 5 units on the horizontal axis and 2 cm to represent 10 units on the vertical axis.

**Required To Plot:** The cumulative frequency curve of the scores.

**Solution:**

![Cumulative Frequency Curve of Scores](image)

(iii) **Required To Find:** Median score.

**Solution:**

From the cumulative frequency curve, the median score corresponds to a cumulative frequency value of \( \frac{1}{2} (100) = 50 \) and reads as 36 on the horizontal axis.

\[ \therefore \text{Median score} = 36. \]
(iv) **Required To Calculate:** Probability a randomly chosen student has a score greater than 40.

**Solution:**

\[
P(\text{student chosen at random scores } > 40) = \frac{\text{No. of students scoring } > 40}{\text{Total no. of students}}
\]

\[
= \frac{21 + 11}{\sum f = 100}
\]

\[
= \frac{32}{100}
\]

\[
= \frac{8}{25}
\]

7. a. **Data:** Prism of cross-sectional area 144 cm² and length 30 cm.

(i) **Required To Calculate:** Volume of the prism.

**Calculation:**

Volume of prism = Area of cross-section \(\times\) Length

\[
= 144 \times 30 \text{ cm}^3
\]

\[
= 4320 \text{ cm}^3
\]

(ii) **Required To Calculate:** Total surface area of the prism.

**Calculation:**

Cross-section is a square of area 144 cm².

\[
\therefore \text{Length } = \sqrt{144} \text{ cm}^2
\]

\[
= 12 \text{ cm}
\]
Area of front and back faces = $144 \times 2$
$= 288 \text{ cm}^2$

Area of L.H.S and R.H.S. rectangular faces = $2(12 \times 30)$
$= 720 \text{ cm}^2$

Area of top and base rectangular faces = $2(12 \times 30)$
$= 720 \text{ cm}^2$

Total surface area of the prism = $288 + 720 + 720$
$= 1728 \text{ cm}^2$

b. Data:

![Diagram of MON sector]

$MON$ is a sector of a circle of radius 15 cm and $MÔN = 45^\circ$.

(i) Required To Calculate: Length of minor arc $MN$.

Calculation:

Length of arc $MN = \frac{45}{360} \times 2\pi(15)$
$= 11.772$

$= 11.78 \text{ cm to 2 decimal places}$

OR

Using
$s = r\theta$
$s = \text{arc length, } r = \text{radius and } \theta = \text{angle in radians}$
$s = (15)(0.785) = 11.775$

$s = 11.78 \text{ cm to 2 decimal places}$
(ii) **Required To Calculate:** Perimeter of figure MON.

**Calculation:**

Perimeter of MON = Arc length MON + Length of radius OM + Length of radius ON

= 11.775 + 15 + 15

= 41.775

= 41.78 to 2 decimal places

(iii) **Required To Calculate:** Area of figure MON.

**Calculation:**

Area of sector MON = \( \frac{45}{360} \pi (15)^2 \)

= 88.312

= 88.31 cm\(^2\) to 2 decimal places

OR

Area of sector = \( \frac{1}{2} r^2 \theta \)

= \( \frac{1}{2} (15)^2 (0.785) \)

= 88.312

= 88.31 cm\(^2\) to 2 decimal places
8. **Data:** Table showing the subdivision of an equilateral triangle.  
**Required To Complete:** The table given.

**Solution:**

<table>
<thead>
<tr>
<th>$n$</th>
<th>Result of each step</th>
<th>No. of triangles formed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td><img src="image" alt="Diagram" /></td>
<td>$1 = 4^0$</td>
</tr>
<tr>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
<td>$4 = 4^1$</td>
</tr>
<tr>
<td>2</td>
<td><img src="image" alt="Diagram" /></td>
<td>$16 = 4^2$</td>
</tr>
<tr>
<td>3</td>
<td><img src="image" alt="Diagram" /></td>
<td>(i) $64 = 4^3$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>6</td>
<td><img src="image" alt="Diagram" /></td>
<td>(ii) $4096 = 4^6$</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>(iii) 8</td>
<td><img src="image" alt="Diagram" /></td>
<td>65536 = 4^8</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
<tr>
<td>$m$</td>
<td><img src="image" alt="Diagram" /></td>
<td>(iv) $4^m$</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
4 & \mid 65536 \\
4 & \mid 16384 \\
4 & \mid 4096 \\
4 & \mid 1024 \\
4 & \mid 256 \\
4 & \mid 64 \\
4 & \mid 16 \\
4 & \mid 4 \\
4 & \mid 1 \\
\end{align*} \]
Section II

9. a. **Required To Factorise:** (i) \(2p^2 - 7p + 3\), (ii) \(5p + 5q + p^2 - q^2\)

Factorising:

(i) \(2p^2 - 7p + 3 = (2p - 1)(p - 3)\)

(ii) \(5p + 5q + p^2 - q^2 = 5(p + q) + (p - q)(p + q) = (p + q)[5 + (p - q)] = (p + q)(5 + p - q)\)

b. **Required To Expand:** \((x + 3)^2(x - 4)\)

Solution:
Expanding
\[
(x + 3)^2(x - 4) = (x + 3)(x + 3)(x - 4)
= (x^2 + 3x + 3x + 9)(x - 4)
= (x^2 + 6x + 9)(x - 4)
= x^3 + 6x^2 + 9x - 4x^2 - 24x - 36
= x^3 + 2x^2 - 15x - 36
\]
Hence, \((x + 3)^2(x - 4) = x^3 + 2x^2 - 15x - 36\), in descending powers of \(x\).

c. **Data:** \(f(x) = 2x^2 + 4x - 5\)

(i) **Required To Express:** \(f(x) = 2x^2 + 4x - 5\) in the form \(a(x + b)^2 + c\).

Solution:

\[
f(x) = 2x^2 + 4x - 5 = 2(x^2 + 2x) - 5
\]

(Half the coefficient of \(x\) is \(\frac{1}{2}(2) = 1\))

Hence \(f(x) = 2x^2 + 4x - 5\)
\[
= 2(x + 1)^2 + * = 2(x^2 + 2x + 1) + * = 2x^2 + 4x + 2\quad \text{(Hence } * = -7)\]
\[
-7
\]
\[
-5
\]

\(\therefore 2x^2 + 4x - 5 \equiv 2(x + 1)^2 - 7\) is of the form \(a(x + b)^2 + c\) where
\( a = 2 \in \mathbb{R} \)
\( b = 1 \in \mathbb{R} \)
\( c = -7 \in \mathbb{R} \)

OR

\[
2x^2 + 4x - 5 = a(x + 5)^2 + c \\
= a(x^2 + 2bx + b^2) + c \\
= ax^2 + 2abx + ab^2 + c
\]

Equating coefficient of \( x^2 \).
\( a = 2 \in \mathbb{R} \)
Equating coefficient of \( x \).
\( 2(2)b = 4 \)
\( b = 1 \in \mathbb{R} \)
Equating constants.
\( 2(1)^2 + c = -5 \)
\( c = -7 \in \mathbb{R} \)
\[
\therefore 2x^2 + 4x - 5 \equiv 2(x + 1)^2 - 7
\]

(ii) \textbf{Required To Find:} The equation of the axis of symmetry.
\textbf{Solution:}
If \( y = ax^2 + bx + c \) is any quadratic curve, the axis of symmetry has equation \( x = \frac{-b}{2a} \).
The equation of the axis of symmetry in the quadratic curve
\( f(x) = 2x^2 + 4x - 5 \) is \( x = \frac{-4}{2(2)} \)
\( x = -1 \)

(iii) \textbf{Required To Find:} Coordinates of the minimum point on the curve.
\textbf{Solution:}
\[
\begin{align*}
    f(x) &= 2x^2 + 4x - 5 \\
    &= 2(x + 1)^2 - 7 \\
2(x + 1)^2 &\geq 0 \quad \forall x \\
\therefore f(x)_{\text{min}} &= -7 \text{ at } 2(x + 1)^2 = 0 \\
&\quad \quad x = -1
\end{align*}
\]
(iv) – (v)

**Required To Draw:** The graph of \( f(x) \) showing the minimum point and the axis of symmetry.

**Solution:**

![Graph of \( f(x) \)](image)

10. **Data:** Pam must buy \( x \) pens and \( y \) pencils.
   a. (i) **Data:** Pam must buy at least 3 pens.
   **Required To Find:** An inequality to represent the above information.
   **Solution:**
   No. of pens bought = \( x \)
   No. of pens is at least 3.
   \( \therefore x \geq 3 \)

   (ii) **Data:** Total number of pens and pencils must not be more than 10.
   **Required To Find:** An inequality to represent the above information.
   **Solution:**
   No. of pencils = \( y \)
   Total number of pens and pencils = \( x + y \)
   \( \therefore (x + y) \) is not more than 10.
   \( \therefore x + y \) is less than or equal to 10.
   \( x + y \leq 10 \)

   (iii) **Data:** \( 5x + 2y \leq 35 \)
   **Required To Find:** Information represented by this inequality.
   **Solution:**
   \( 5x + 2y \leq 35 \) (data)
   \( 5x \) represents the cost of \( x \) pens at $5.00 each and \( 2y \) represents the cost of \( y \) pencils at $2.00 each.
   Total cost is \( 5x + 2y \).
Since \( 5x + 2y \leq 35 \), then the total cost of \( x \) pens and \( y \) pencils is less than or equal to $35.00.
That is, the total cost of the \( x \) pens and \( y \) pencils is not more than $35.00.

b. (i) **Required To Draw:** The graphs of the two inequalities obtained on answer sheet.

**Solution:**
The line \( x = 3 \) is a vertical line.
The region \( x \geq 3 \) is

Obtaining 2 points on the line \( x + y = 10 \)
When \( x = 0 \)
\[ 0 + y = 10 \]
\[ y = 10 \]
The line \( x + y = 10 \) passes through the point \((0, 10)\).
When \( y = 0 \)
\[ x + 0 = 10 \]
\[ x = 10 \]
The line \( x + y = 10 \) passes through the point \((10, 0)\).

The region with the smaller angle satisfies the \( \leq \) region.
The region with satisfies \( x + y \leq 10 \) is
The graph of the line \( x + y = 10 \) was given. It passes through the points (7, 0) and (3, 10).

The region with the smaller angle satisfies the \( \leq \) region. The region which satisfies \( 5x + 2y \leq 35 \) is

The line \( y = 0 \) is the horizontal \( x \)-axis. The region which satisfies \( y \geq 0 \) is
The region which satisfies all four inequalities is the area in which all four previously shaded regions overlap. The region which satisfies all four inequalities is
(ii) **Required To Find:** The vertices of the region bounded by the 4 inequalities is shown ABCD (the feasible region)

**Solution:**

\[ A (3, 0) \quad B (3, 7) \quad C (5, 5) \quad D (7, 0) \]
c. **Data:** A profit of $1.50 is made on each pen and a profit of $1.00 is made on each pencil.

(i) **Required To Find:** The profit in terms of $x$ and $y$.

**Solution:**

Let the total profit on pens and pencils be $P$. The profit on $x$ pens at $\frac{1}{2}$ and $y$ pencils at $1$ each = \( x \times \frac{1}{2} + y \times 1 \)

\[ P = \frac{1}{2}x + y \]

(ii) **Required To Find:** Maximum profit.

**Solution:**

Choosing only $B (3, 7)$, $C (5, 5)$ and $D (7, 0)$.

At $B$ $x = 3$ $y = 7$

\[ P = 3 \left( \frac{1}{2} \right) + 7 \]

= $11\frac{1}{2}$

= $11.50$

At $C$ $x = 5$ $y = 5$

\[ P = 5 \left( \frac{1}{2} \right) + 5 \]

= $12\frac{1}{2}$

= $12.50$

At $D$ $x = 7$ $y = 0$

\[ P = 7 \left( \frac{1}{2} \right) \]

= $10\frac{1}{2}$

= $10.50$

\[ \therefore \text{Maximum profit made is } $12.50 \text{ when Pam buys 5 pens and 5 pencils}. \]
(iii) Required To Find: The maximum number of pencils Pam can buy if she buys 4 pens.

Solution:

When \( x = 4 \) the maximum value of \( y \in \mathbb{Z}^+ \) is 6. Therefore, when 4 pens are bought, the maximum number of pencils that can be bought that satisfies all conditions is 6.
11. a. **Data:** Diagram showing 2 circles of radii 5 cm and 2 cm touching at \( T \), \( XSRY \) is a straight line touching the circles at \( S \) and \( R \).

(i) (a) **Required To State:** Why \( PTQ \) is a straight line.

**Solution:**
The tangent to both circles at \( T \) is a common tangent.

The tangent makes an angle of 90° with the radius \( PT \) and 90° with the radius \( TQ \).
(Angle made by a tangent to a circle and the radius, at the point of contact = 90°).
\[ \therefore P\hat{Q}T = 180° \] (as illustrated) and \( PTQ \) is a straight line.

(b) **Required To State:** The length of \( PQ \).

**Solution:**
Length of \( PQ \) = Length of \( PT \) + Length of \( TQ \)
\[ = 5 + 2 \]
\[ = 7 \text{ cm} \]

(c) (i) **Required To State:** Why \( PS \) is parallel to \( QR \).

**Solution:**
\[ P\hat{S}R = Q\hat{S}R = 90° \]
(Angle made by a tangent to a circle and the radius, at the point of contact = 90°).

There are corresponding angles, when $PS$ is parallel to $QR$ and $SR$ is a transversal.

(ii) **Data:** $N$ is a point such that $QN$ is perpendicular to $PS$.

![Diagram](image)

(a) **Required To Calculate:** The length $PN$.
**Calculation:**
$QRSN$ is a rectangle and hence $NS = 2$ cm, $PS = 5$ cm
\[
\therefore PN = 5 - 2 = 3 \text{ cm}
\]

(b) **Required To Calculate:** The length $SR$.
**Calculation:**
\[
NQ = \sqrt{(7)^2 - (3)^2} = \sqrt{40} \text{ cm}
\]
\[
SR = NQ = \sqrt{40} \text{ cm exactly}
\]
\[
= 6.324 \text{ cm}
\]
\[
= 6.32 \text{ cm to 2 decimal places}
\]
b. **Data:** Circle, centre $O$ and $MOL = 110^\circ$.

![Diagram of a circle with points M, O, L and angles](image)

(i) **Required To Calculate:** $MNL$

**Calculation:**

$$MNL = \frac{1}{2} (110^\circ)$$

$$= 55^\circ$$

(Angle subtended by a chord at the centre of a circle is twice the angle it subtends at the circumference, standing on the same arc).

(ii) **Required To Calculate:** $LMO$

**Calculation:**

$$OM = OL \quad \text{(radii)}$$

$$LMO = \frac{180^\circ - 110^\circ}{2}$$

$$= 35^\circ$$

(Base angles of an isosceles triangle are equal and sum of the angles in a triangle = $180^\circ$).

12. **Data:** The distances and directions of a boat traveling from A to B and then to C.

a. **Required To Draw:** Diagram of the information given, showing the north direction, bearings $135^\circ$ and $060^\circ$ and distances $8$ km and $15$ km.

**Solution:**

![Diagram of the boat's journey](image)
b. (i) **Required To Calculate:** The distance AC.

**Calculation:**

\[ AB^2 = 45^\circ + 60^\circ \]
\[ = 105^\circ \]
\[ AC^2 = (15)^2 + (8)^2 - 2(15)(8)\cos 105^\circ \quad \text{(cosine law)} \]
\[ = 18.738 \text{ km} \]
\[ = 18.74 \text{ km to 2 decimal places} \]

(ii) **Required To Calculate:** \( B\hat{C}A \)

**Calculation:**

Let \( B\hat{C}A = \theta \)

\[ \frac{15}{\sin \theta} = \frac{18.738}{\sin 105^\circ} \quad \text{(sine law)} \]

\[ \therefore \sin \theta = \frac{15 \sin 105^\circ}{18.738} \]
\[ = 0.7732 \]

\[ \therefore \theta = \sin^{-1}(0.7732) \]
\[ \theta = 50.64^\circ \]
\[ = 50.6^\circ \text{ to the nearest } 0.1^\circ \]

(iii) **Required To Calculate:** The bearing from \( A \) from \( C \).

**Calculation:**

The bearing of \( A \) from \( C \) = \( 180^\circ + 60^\circ + 50.64^\circ \)
\[ = 290.64^\circ \]
\[ = 290.6^\circ \text{ to the nearest } 0.1^\circ \]
13. **Data:** Vector diagram with $\overrightarrow{OP} = \vec{r}$, $\overrightarrow{PM} = \vec{s}$ and $OMN$ a straight line with midpoint $M$.

$\overrightarrow{OX} = \frac{1}{3} \overrightarrow{OM}$ and $\overrightarrow{PX} = 4\overrightarrow{XQ}$

a. **Required To Sketch:** Diagram illustrating the information given.

**Solution:**

![Diagram](image)

b. (i) **Required To Express:** $\overrightarrow{OM}$ in terms of $\vec{r}$ and $\vec{s}$.

**Solution:**

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$

$$= \vec{r} + \vec{s}$$

(ii) **Required To Express:** $\overrightarrow{PX}$ in terms of $\vec{r}$ and $\vec{s}$.

**Solution:**

$$\overrightarrow{OX} = \frac{1}{3} \overrightarrow{OM}$$

$$= \frac{1}{3}(\vec{r} + \vec{s})$$

$$\overrightarrow{PX} = \overrightarrow{PO} + \overrightarrow{OX}$$

$$= -\vec{r} + \frac{1}{3}(\vec{r} + \vec{s})$$

$$= -\frac{2}{3}\vec{r} + \frac{1}{3}\vec{s}$$

(iii) **Required To Express:** $\overrightarrow{OM}$ in terms of $\vec{r}$ and $\vec{s}$.

**Solution:**

$$\overrightarrow{PX} = 4\overrightarrow{XQ}$$

$$\overrightarrow{PQ} = \frac{5}{4} \overrightarrow{PX}$$

$$= \frac{5}{4} \left( -\frac{2}{3}\vec{r} + \frac{1}{3}\vec{s} \right)$$
\[ \overrightarrow{PQ} = -\frac{5}{6}r + \frac{5}{12}s \]
\[ \overrightarrow{QM} = \overrightarrow{QP} + \overrightarrow{PM} \]
\[ = -\left( -\frac{5}{6}r + \frac{5}{12}s \right) + s \]
\[ = \frac{5}{6}r + \frac{7}{12}s \]

c. **Required To Prove:** \( \overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP} \)

**Proof:**
\[ 2\overrightarrow{PM} = 2(\overrightarrow{s}) \]
\[ \overrightarrow{OP} = r \]
\[ 2\overrightarrow{PM} + \overrightarrow{OP} = 2\overrightarrow{s} + \overrightarrow{r} \]
\[ = \overrightarrow{r} + 2\overrightarrow{s} \]
Hence, \( \overrightarrow{PN} = 2\overrightarrow{PM} + \overrightarrow{OP} \) \((= \overrightarrow{r} + 2\overrightarrow{s})\)

Q.E.D

14. a. **Data:** \( D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix} \)

**Required To Calculate:** \( p \)

**Calculation:**
If \( D = \begin{pmatrix} 1 & 9p \\ p & 4 \end{pmatrix} \) is singular then \( \det D = 0 \).
\[ \therefore (1 \times 4) - (9p \times p) = 0 \]
\[ 4 = 9p^2 \]
\[ p^2 = \frac{4}{9} \]
\[ p = \pm \frac{2}{3} \]

Hence, \( p = \pm \frac{2}{3} \).
b. **Data:** $2x + 5y = 6$ and $3x + 4y = 8$

(i) **Required To Express:** The above equations in the form $AX = B$.

**Solution:**

$2x + 5y = 6$

$3x + 4y = 8$

Hence, \[
\begin{pmatrix}
2 & 5 \\
3 & 4
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
= 
\begin{pmatrix}
6 \\
8
\end{pmatrix}
\]...matrix equation

is of the form $AX = B$ where

$A = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

$X = \begin{pmatrix} x \\ y \end{pmatrix}$ and

$B = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$ are matrices.

(ii) (a) **Required To Calculate:** Determinant of $A$.

**Calculation:**

$\det A = (2 \times 4) - (5 \times 3)$

$= 8 - 15$

$= -7$

(b) **Required To Prove:** $A^{-1} = \begin{pmatrix}
-\frac{4}{7} & \frac{5}{7} \\
\frac{3}{7} & -\frac{2}{7}
\end{pmatrix}$.

**Proof:**

$A^{-1} = -\frac{1}{7}\begin{pmatrix}
4 & -(5) \\
-(3) & 2
\end{pmatrix}$

$= \begin{pmatrix}
-\frac{4}{7} & \frac{5}{7} \\
\frac{3}{7} & -\frac{2}{7}
\end{pmatrix}$

Q.E.D.
(c) **Required To Calculate:** \( x \) and \( y \\

**Calculation:**

\[
AX = B
\]

\[
\times A^{-1}
\]

\[
A \times A^{-1} \times X = A^{-1} \times B
\]

\[
I \times X = A^{-1} B
\]

\[
X = A^{-1} B
\]

and

\[
\begin{pmatrix}
  x \\
  y
\end{pmatrix} = \begin{pmatrix}
  -\frac{4}{7} & \frac{5}{7} \\
  \frac{3}{7} & -\frac{2}{7}
\end{pmatrix} \begin{pmatrix}
  6 \\
  8
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  \left(-\frac{4}{7} \times 6\right) + \left(\frac{5}{7} \times 8\right) \\
  \left(\frac{3}{7} \times 6\right) + \left(-\frac{2}{7} \times 8\right)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  -\frac{24}{7} + \frac{40}{7} \\
  \frac{18}{7} - \frac{16}{7}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
  2 \frac{2}{7} \\
  2 \frac{2}{7}
\end{pmatrix}
\]

Equating corresponding \( x = 2 \frac{2}{7} \) and \( y = 2 \frac{2}{7} \).