CSEC JANUARY 2008 MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. (i) Required To Calculate: \( \frac{1 - \frac{3}{7}}{\frac{1}{2} \times \frac{1}{5}} \)

Calculation:
Numerator:
\[
1 - \frac{3}{7} = \frac{8}{7} - \frac{3}{4} = \frac{4(8) - 7(3)}{28} = \frac{32 - 21}{28} = \frac{11}{28}
\]

Denominator:
\[
\frac{1}{2} \times \frac{1}{5} = \frac{5}{2} \times \frac{1}{5} = \frac{1}{2}
\]

Hence,
\[
\frac{1 - \frac{3}{7}}{\frac{1}{2} \times \frac{1}{5}} = \frac{\frac{11}{28}}{\frac{1}{2}} = \frac{11}{14} \text{ (in exact form)}
\]

(ii) Required To Calculate: \( 2 - \frac{0.24}{0.15} \)

Calculation:
\[
2 - \frac{0.24}{0.15} = 2 - 1.6 = 0.4 \text{ (in exact form)}
\]
b. **Data:** Bicycle with prices for cash or hire purchase (H.P.) payments.
   
   (i) **Required To Calculate:** Total hire purchase price for the bicycle.
   
   **Calculation:**
   
   Hire purchase price of the bicycle = Deposit + Total for 10 installments
   
   \[= 69.00 + 10(28.50)\]
   
   \[= 354.00\]
   
   (ii) **Required To Calculate:** Difference between hire purchase price and cash price.
   
   **Calculation:**
   
   Difference between the hire purchase price and the cash price
   
   \[= 354.00 - 319.95\]
   
   \[= 34.05\]
   
   (iii) **Required To Express:** Difference between 2 prices as a percentage of the cash price.
   
   **Solution:**
   
   The difference in price as a percentage of the cash price
   
   \[\left(\frac{34.05}{319.95} \times 100\right)\%\]
   
   \[= 10.642\%\]
   
   \[= 10.64\% (to 2 decimal places)\]

2. a. (i) **Required To Solve:** \(3 - 2x < 7\).
   
   **Solution:**
   
   \[3 - 2x < 7\]
   
   \[-2x < 7 - 3\]
   
   \[-2x < 4\]
   
   \[\times -1\]
   
   \[2x > -4\]
   
   \[\div 2\]
   
   \[x > -2\]
   
   (ii) **Required To Determine:** Smallest value of \(x\) that satisfies the above inequality.
   
   **Solution:**
   
   \(W = \{0, 1, 2, 3, ...\}\)
   
   If \(x \in W\), then the smallest \(x\) is 0.
b. **Required To Factorise:** *(i) \(x^2 - xy, (ii) a^2 - 1, (iii) 2p - 2q - p^2 + pq\)*

**Solution:**

(i) \(x^2 - xy = x \cdot x - x \cdot y = x(x - y)\)

(ii) \(a^2 - 1 = (a)^2 - (1)^2\)

This is a difference of 2 squares

And \(a^2 - 1 = (a - 1)(a + 1)\)

(iii) \(2p - 2q - p^2 + pq = 2(p - q) - p(p - q) = (p - q)(2 - p)\)

c. **Data:** Table showing types of cakes, cost and the number sold.

(i) **Required To Find:** An expression, in terms of \(k\), for the amount of money collected for the sale of sponge cakes.

**Solution:**

The cost of 2 sponge cakes at \$(k + 5) each = 2 \times (k + 5) = $(2k + 10)

(ii) **Required To Find:** An expression in terms of \(k\), for the total amount of collected.

**Solution:**

Cost of sponge cakes = \$(2k + 10)

Cost of 10 chocolate cakes at \$k each = k \times 10 = $10k

Cost of 4 fruit cakes at \$2k each = 2k \times 4 = $8k

Total amount of money collected = \((2k + 10) + 10k + 8k\) = \$(20k + 10)

(iii) **Data:** Total amount of money collected = \$140.00

**Required To Calculate:** \(k\)

**Calculation:**

Total amount of money collected = \$140.00

\[ \therefore 20k + 10 = 140 \]

\[ 20k = 140 - 10 \]

\[ 20k = 130 \]

\[ k = \frac{130}{20} \]

\[ k = 6.5 \]
3. a. **Data:** Given the elements of universal set, $U$, $S$ and $T$.

   (i) **Required To Draw:** Venn diagram to represent the information given.
   
   **Solution:**
   
   $S = \{k, l, m, p\}$
   
   $T = \{k, p, q\}$
   
   $S \cap T = \{k, p\}$

   ![Venn diagram](image)

   (ii) (a) **Required To List:** Members of $S \cup T$.
   
   **Solution:**
   
   $S \cup T = \{l, m, k, p, q\}$ as shown.

   (b) **Required To List:** Members of $S'$.
   
   **Solution:**
   
   $S' = \{r, q, n\}$ as shown.

b. **Data:** Quadrilateral $ABCD$ with $AB = AD$, $B\hat{C}D = 90^\circ$, $D\hat{B}C = 42^\circ$ and $AB$ parallel to DC.

   ![Quadrilateral](image)

   (i) **Required To Calculate:** $A\hat{B}C$
   
   **Calculation:**
   
   $A\hat{B}C = 180^\circ - 90^\circ$
   
   $= 90^\circ$

   (Co-interior angles are supplementary).
(ii) **Required To Calculate:** \( \hat{ABD} \)
**Calculation:**
\[
\hat{ABD} = 90^\circ - 42^\circ \\
= 48^\circ
\]

(iii) **Required To Calculate:** \( \hat{BAD} \)
**Calculation:**
\[
\hat{AB} = \hat{AD} \quad \text{(data)} \\
\therefore \hat{ABD} = 48^\circ \\
\text{(Base angles of an isosceles triangle are equal).} \\
\hat{BAD} = 180^\circ - (48^\circ + 48^\circ) \\
= 84^\circ \\
\text{(Sum of angles in a triangle = 180\(^\circ\)).}
\]

4. a. **Data:** John left Port A at 07:30 hours and travelled to Port B in the same time zone.
He arrives at Port B at 14:20 hours.

(i) **Required To Find:** Duration of the journey.
**Solution:**
Departure time from A is 07:30 hours.
Arrival time at B is 14:20 hours.
Duration of the journey = 14:20 - 07:30
\[
= 6:50
\]
\[
= 6 \text{ hours} 50 \text{ minutes}
\]
\[
= 6\frac{5}{6} \text{ hours}
\]

(ii) **Data:** John travelled 410 km.
**Required To Calculate:** Average speed.
**Calculation:**
Average speed = \( \frac{\text{Total distance covered}}{\text{Total time taken}} \)
\[
= \frac{410\text{km}}{6\frac{5}{6} \text{ hours}} \\
= 60 \text{ kmh}^{-1}
\]

b. **Data:** Diagram showing a circle of radius 3.5 cm, centre \( O \) and square \( OPQR \).
(i) **Required To Calculate:** Area of the circle.
**Calculation:**
\[
\text{Area of circle} = \frac{22}{7} (3.5)^2
= 38.5 \text{ cm}^2
\]

(ii) **Required To Calculate:** Area of square $OPQR$.
**Calculation:**
\[
\text{Area of square } OPQR = 3.5 \times 3.5
= 12.25 \text{ cm}^2
\]

(iii) **Required To Calculate:** Area of the shaded region.
**Calculation:**
\[
\text{Area of the shaded region} = \text{Area of square } OPQR - \text{Area of sector } OPR.
= 12.25 - \frac{1}{4} (38.5)
= 2.625 \text{ cm}^2
= 2.63 \text{ cm}^2 \text{ (to 2 decimal places)}
\]

5. **Data:** Bar graph illustrating the number of books read by the boys of a book club.
   a. **Required To Draw:** The frequency data representing the information given.
   **Solution:**
   From the bar graph
   
<table>
<thead>
<tr>
<th>No. of books read, $x$</th>
<th>No. of boys, $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$\sum f = 36$</td>
</tr>
</tbody>
</table>

   b. **Required To Find:** The number of boys in the book club.
Solution:
No. of boys in the book club = \( \sum f \)
= 36

c. Required To Find: Modal number of books read.
Solution:
Modal number of books read = 2, as \( x = 2 \) corresponds to the maximum frequency, 17.

d. Required To Calculate: Total number of books read.
Calculation:
Total number of books read = \((2 \times 0) + (6 \times 1) + (17 \times 2) + (8 \times 3) + (3 \times 4)\)
= 6 + 34 + 24 + 12
= 76

e. Required To Calculate: The mean number of books read.
Calculation:
The mean number of books read, \( \bar{x} \).
\[
\bar{x} = \frac{\sum f\bar{x}}{\sum f}
\]
\[
= \frac{76}{36}
\]
\[
= 2.1
\]
This value is a whole number and may be represented by the integer 2.

f. Required To Calculate: Probability a randomly chose boy reads at least books.
Calculation:
\[
P(\text{Randomly chosen boy reads } \geq 3 \text{ books}) = \frac{\text{No. of boys reading } \geq 3 \text{ books}}{\text{Total no. of boys}}
\]
\[
= \frac{8 + 3}{\sum f = 36}
\]
\[
= \frac{11}{36}
\]

6. a. Data: Diagram showing a pattern of congruent right angled triangles as
(i) **Required To Describe:** Transformation that maps \( \triangle BCL \) onto \( \triangle FHL \).

**Solution:**

\( \triangle BCL \rightarrow \triangle HFL \)

\( L \) is a common point.

Image is re-oriented and congruent.

\( BLF \) is a straight line.

\( \therefore BCL \) is mapped onto \( HFL \) by a rotation of \( 180^\circ \) (clockwise or anti-clockwise) about \( L \).

(ii) **Required To Describe:** Transformation that maps \( \triangle BCL \) onto \( \triangle HFG \).

**Solution:**

\( \triangle BCL \) and \( \triangle HFG \) have the same ‘orientation’.

\( \triangle BCL \) is mapped onto \( \triangle HFG \) by a horizontal shift 4 units to the right and 1 unit vertically downwards. This may be represented by the translation, \( T \)

where

\[
T = \begin{pmatrix} 4 \\ -1 \end{pmatrix}.
\]

b. (i) **Required To Construct:** Parallelogram \( WXYZ \), in which, \( WX = 7.0 \text{ cm}, WZ = 5.5 \text{ cm} \) and \( \angle WZX = 60^\circ \).

**Solution:**
(ii) **Required To Find:** The length of $WY$.
**Solution:**
$WY = 10.9$ cm (by measurement.)

7. **Data:** Incomplete table for $y = x^2 - 4x$.
   a. **Required To Complete:** The table of values for $y = x^2 - 4x$.
   **Solution:**
   
<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>5</td>
<td>0</td>
<td>-3</td>
<td>-4</td>
<td>-3</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

When $x = 0$

$y = (0)^2 - 4(0)$

$= 0$

When $x = 3$

$y = (3)^2 - 4(3)$

$= -3$

When $x = 5$

$y = (5)^2 - 4(5)$

$= 5$
b. **Required To Draw:** The graph of \( y = x^2 - 4x \).

**Solution:**

![Graph of \( y = x^2 - 4x \)]

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c. (ii) **Required To Find:** Points at which the curve meets the line.

**Solution:**

The line \( y = 2 \) and the curve \( y = x^2 - 4x \) meet at \( x = -0.5 \) and \( x = 4.5 \).

(iii) **Required To Find:** The equation whose roots are the coordinates found above.

**Solution:**

\( x = -0.5 \) and \( x = 4.5 \) are the roots of the equation \( x^2 - 4x = 2 \) or \( x^2 - 4x - 2 = 0 \).
8. a. **Data:** Table showing the sum of rational numbers.

**Required To Complete:** The table given.

<table>
<thead>
<tr>
<th>n</th>
<th>SERIES</th>
<th>SUM</th>
<th>FORMULA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>(\frac{1}{2}(1)(1+1))</td>
</tr>
<tr>
<td>2</td>
<td>1 + 2</td>
<td>3</td>
<td>(\frac{1}{2}(2)(2+1))</td>
</tr>
<tr>
<td>3</td>
<td>1 + 2 + 3</td>
<td>6</td>
<td>(\frac{1}{2}(3)(3+1))</td>
</tr>
<tr>
<td>4</td>
<td>1 + 2 + 3 + 4</td>
<td>10</td>
<td>(\frac{1}{2}(4)(4+1))</td>
</tr>
<tr>
<td>5</td>
<td>1 + 2 + 3 + 4 + 5</td>
<td>15</td>
<td>(\frac{1}{2}(5)(5+1))</td>
</tr>
<tr>
<td>(i) 6</td>
<td>1 + 2 + 3 + 4 + 5 + 6</td>
<td>21</td>
<td>(\frac{1}{2}(6)(6+1))</td>
</tr>
</tbody>
</table>

\[\vdots\] \[\vdots\] \[\vdots\]

| 8  | 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8      | 36   | \(\frac{1}{2}(8)(8+1)\) |

\[\vdots\] \[\vdots\] \[\vdots\]

| (ii) n | 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + …+ \(n\) | \(\frac{1}{2} n(n+1)\) |

The formula for the sum is a constant, \(\frac{1}{2} n(n+1)\), example

When \(n = 5\) \[\text{Sum is } \frac{1}{2}(5)(5+1) = 15\]

\[\therefore \text{Sum} = 15\]

When \(n = 6\) \[\text{Sum} = \frac{1}{2}(6)(6+1) = 21\]

\[\therefore \text{Sum} = 21\]

For (ii), the formula is \(\frac{1}{2} n(n+1)\).

b. **Data:**

\[1 + 2 + 3 = 6 \text{ and } 1^3 + 2^3 + 3^3 = 6^2 = 36\]

So too,

\[1 + 2 + 3 + 4 = 10 \text{ and } 1^3 + 2^3 + 3^3 + 4^3 = 10^2 = 100\]
(i) **Required To Find:** \(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3\)

**Solution:**
\[
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36
\]
From the table
\[
1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 = 36^2
= 1296
\]

(ii) **Required To Find:** \(1^3 + 2^3 + 3^3 + \ldots + n^3\)

**Solution:**
\[
1 + 2 + 3 + \ldots + n = \frac{1}{2}n(n + 1)
\]
\[
\therefore 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{1}{2}n(n + 1)\right)^2
\]

c. **Required To Find:** \(1^3 + 2^3 + 3^3 + 4^3 + \ldots + 12^3\)

**Solution:**
\[
1 + 2 + 3 + 4 + \ldots + 12 = \frac{1}{2}(12)(12 + 1)
= 78
\]
\[
\therefore 1^3 + 2^3 + 3^3 + 4^3 = (78)^2
= 6084
\]

9. a. **Data:** Volume, \(V\) of a gas varies inversely as the pressure, \(P\), with temperature constant.

(i) **Required To Find:** Equation relating \(V\) to \(P\).

**Solution:**
\[
V \propto \frac{1}{P}
\]
\[
V = k \times \frac{1}{P}
\]
\((k\) is constant of proportion or variation\)

(ii) **Required To Find:** \(k\) when \(V = 12.8\) and \(P = 500\).

**Solution:**
\[
V = 12.8\text{ when } P = 500
\]
\[
12.8 = k \times \frac{1}{500}
\]
\[
k = 12.8 \times 500
= 6400
\]
(iii) **Required To Calculate:** \( V \) when \( P = 480 \).

**Calculation:**

When

\( P = 480 \)

\[
V = 6400 \times \frac{1}{480}
\]

\[
= 13 \frac{1}{3}
\]

b. **Data:** Right-angled triangle of sides \( a, (a - 7) \) and \( (a + 1) \).

(i) **Required To Find:** An equation in terms of \( a \) to relate the three sides, using Pythagoras’ Theorem.

**Solution:**

\[
(a)^2 + (a - 7)^2 = (a + 1)^2
\]

Pythagoras’Theorem

\[
a^2 + a^2 - 14a + 49 = a^2 + 2a + 1
\]

\[
a^2 - 16a + 48 = 0
\]

(ii) **Required To Calculate:** \( a \)

**Calculation:**

\[
a^2 - 16a + 48 = 0
\]

\[
(a - 12)(a - 4) = 0
\]

\[
a = 4 \text{ or } 12
\]

If \( a = 4 \), the side \( (a - 7) \) would compute to be a negative value. Hence, \( a = 12 \) only.

(iii) **Required To State:** The lengths of the 3 sides of the triangle.

**Solution:**

The lengths of the 3 sides are
10. a. **Data:** School buys \( x \) balls and \( y \) bats.

(i) **Required To Find:** Inequality for the information given.

**Solution:**
Total number of balls and bats is no more than 30.

\[ x + y \leq 30 \]

Hence, \( x + y \leq 30 \) ...(1)

(ii) **Data:** School allows no more than $360 to be spent on bats and balls. The cost of a ball is $6 and the cost of a bat is $24.

**Required To Find:** Inequality to represent the information given.

**Solution:**
The cost of \( x \) balls at $6 each and \( y \) bats at $24 each is
\[ (x \times 6) + (y \times 24) = 6x + 24y \]

Budget allows no more than $360. Similarly,
\[ 6x + 24y \leq 360 \]

\[ \div 6 \]
\[ x + 4y \leq 60 \] ...(1)

b. **Required To Draw:** The graphs of the inequalities shown above, shade the region that satisfies the inequalities and state the vertices of the feasible region.

**Solution:**
\( x \geq 0 \) and \( y \geq 0 \) identifies the 1st quadrant.

Obtaining 2 points on the line \( x + y = 30 \)

When \( x = 0 \)
\[ 0 + y = 30 \]
\[ y = 30 \]

The line \( x + y = 30 \) passes through the point (0, 30).

When \( y = 0 \)
\[ x + 0 = 30 \]
\[ x = 30 \]

The line \( x + y = 30 \) passes through the point (30, 0).
The region with the smaller angle corresponds to the $\leq$ region.
The region which satisfies $x + y \leq 30$ is

Obtaining 2 points on the line $x + 4y = 60$.
When $x = 0$

$0 + 4y = 60$

$4y = 60$

$y = \frac{60}{4}$

$y = 15$

The line $x + 4y = 60$ passes through the point $(0, 15)$.
When $y = 0$

$x + 4(0) = 60$

$x = 60$

The line $x + 4y = 60$ passes through the point $(60, 0)$.

The region with the smaller angle corresponds to the $\leq$ region.
The region which satisfies $x + 4y \leq 60$ is
The feasible region is the area in which both previously shaded regions overlap.
The vertices of the feasible region are \(O\) \((0, 0)\), \(A\) \((0, 15)\), \(B\) \((20, 10)\) and \(C\) \((30, 0)\).

c. **Data:** Profit made of each ball is $1 and profit made on each bat is $3.
(i) **Required To Find:** The profit for each of the combinations above.

**Solution:**

\[ P = x + 3y \]

Testing the point \((0, 15)\), \((30, 0)\) and \((20, 10)\).

When \(x = 0\) and \(y = 15\)

\[ P = 0 + 3(15) \]

\[ = $45 \]
When \( x = 30 \) and \( y = 0 \)
\[
P = 30 + 3(0)
\]
\[
= 30
\]

When \( x = 20 \) and \( y = 10 \)
\[
P = 20 + 3(10)
\]
\[
= 50
\]

As a point of interest, the only point to be considered should be \((20, 10)\), where \( x = 20 \) and \( y = 10 \) since the question specifically indicates – a school buys \( x \) balls AND \( y \) balls. (They cannot buy 0 bats or 0 balls, that is \( x \) and \( y \in Z^+ \).

(ii) **Required To Find:** Maximum profit that may be made.

**Solution:**
The maximum profit, \( P_{\text{max}} = 50 \), which occurs when \( x = 20 \) and \( y = 10 \).

11. a. **Data:** \( O \) is the centre of the circle \( WXY \) and \( \angle WXY = 50^\circ \)

(i) **Required To Calculate:** \( \angle W\hat{O}Y \)

**Calculation:**
\[
\angle W\hat{O}Y = 2(50^\circ)
\]
\[
= 100^\circ
\]

(The angle subtended by a chord at the centre of the circle is twice the angle subtended at the circumference, standing on the same arc).
(ii) **Required To Calculate: \( \overline{OY} \)**  
**Calculation:**  

\[
\overline{OW} = \overline{OY} \quad \text{(radii)}
\]

\[
\angle \overline{OWY} = \angle \overline{OWY} \text{ (the base angles of an isosceles triangle are equal)}
\]

\[
= \frac{180^\circ - 100^\circ}{2} = 40^\circ
\]

(Sum of the angles in a triangle = 180\(^\circ\)).

b. **Data:** Three buoys \( A, B \) and \( C \), their relative distances apart and their positions.

(i) **Required To Sketch:** Diagram showing the information given.

**Solution:**

![Diagram showing positions of buoys A, B, and C](image)

(ii) **Required To Calculate:** Distance \( AC \).

**Calculation:**  

\[
\angle \overline{ABC} = 90^\circ - 10^\circ = 80^\circ
\]

\[
AC^2 = (125)^2 + (75)^2 - 2(125)(75)\cos 80^\circ \quad \text{(cos law)}
\]

\[
AC = 134.14 \text{ m}
\]

\[
AC = 134.1 \text{ m}
\]
(iii) **Required To Calculate:** Bearing of \( C \) from \( A \).

**Calculation:**

Let \( \angle BAC = \alpha^\circ \)

\[
\frac{75}{\sin \alpha} = \frac{134.1}{\sin 80^\circ} \quad \text{(Sine Law)}
\]

\[
\sin \alpha = \frac{75 \sin 80^\circ}{134.1}
\]

\[
\alpha = 33.4^\circ
\]

The bearing of \( C \) from \( A \) = \( 90^\circ + 33.4^\circ \)

\[= 123^\circ \text{ to the nearest degree} \]

12. a. **Data:** Diagram illustrating a vertical pole and tower standing on horizontal ground.

(i) **Required To Calculate:** Horizontal distance \( AB \).

**Calculation:**

\( \angle DAB = 5^\circ \) (alternate angles).

\[
\tan 5^\circ = \frac{2.5}{AB}
\]

\[
AB = \frac{2.5}{\tan 5^\circ}
\]

\[= 28.57 \text{ m} \]

\[= 28.6 \text{ m (to 1 decimal place)} \]
(ii) **Required To Calculate:** Height of the tower $BC$.

**Calculation:**

$DE = 28.57$

(Opposite sides of a rectangle).

$\tan 20^\circ = \frac{CE}{28.57}$

$CE = 28.57 \tan 20^\circ$

$= 10.39 \text{ m}$

Height of tower $BC = 2.5 + 10.39$

$= 12.89 \text{ m}$

$= 12.9 \text{ m}$

b. No solution has been offered for this question as it is based on latitude and longitude (Earth Geometry) which has been removed from the syllabus.

13. a. **Data:** $P$ and $Q$ are the midpoints of $AB$ and $BC$ of vector triangle $ABC$.

(i) **Required To Sketch:** Diagram to show the information given.

**Solution:**

```
\begin{align*}
\overrightarrow{AB} &= 2x \\
\therefore \overrightarrow{AP} &= \overrightarrow{PB} \\
&= x \\
\overrightarrow{BC} &= 3y \\
\therefore \overrightarrow{BQ} &= \overrightarrow{QC} \\
&= \frac{1}{2} (3y) \\
&= \frac{3}{2} y \\
&= 1 \frac{1}{2} y
\end{align*}
```
(ii) (a) **Required To Find:** Expression in terms of $x$ and $y$ for $\overrightarrow{AC}$.

Solution:

\[
\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}
\]

\[
= 2x + 3y
\]

(b) **Required To Find:** Expression in terms of $x$ and $y$ for $\overrightarrow{PQ}$.

Solution:

\[
\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}
\]

\[
= x + \frac{1}{2} y
\]

(iii) **Required To Prove:** $\overrightarrow{PQ} = \frac{1}{2} \overrightarrow{AC}$

**Proof:**

\[
\overrightarrow{PQ} = x + \frac{1}{2} y
\]

\[
\overrightarrow{AC} = 2x + 3y
\]

\[
= 2 \left( x + \frac{1}{2} y \right)
\]

\[
= 2 \overrightarrow{PQ}
\]

\[
\overrightarrow{PQ} = \frac{1}{2} \overrightarrow{AC}
\]

Q.E.D.

b. **Data:**

\[
\overrightarrow{OR} = \left( \frac{3}{4} \right), \quad \overrightarrow{OS} = \left( -\frac{1}{6} \right) \quad \text{and} \quad \overrightarrow{OT} = \left( \frac{5}{2} \right)
\]
(i) (a) **Required To Express:** \( \overrightarrow{RT} \) in the form \( \left( \begin{array}{c} a \\ b \end{array} \right) \).

**Solution:**
\[
\overrightarrow{RT} = \overrightarrow{RO} + \overrightarrow{OT} \\
= \left( \begin{array}{c} 3 \\ 4 \end{array} \right) + \left( \begin{array}{c} 5 \\ -2 \end{array} \right) \\
= \left( \begin{array}{c} 2 \\ -6 \end{array} \right)
\]

(b) **Required To Express:** \( \overrightarrow{SR} \) in the form \( \left( \begin{array}{c} a \\ b \end{array} \right) \).

**Solution:**
\[
\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR} \\
= \left( \begin{array}{c} -1 \\ 6 \end{array} \right) + \left( \begin{array}{c} 3 \\ 4 \end{array} \right) \\
= \left( \begin{array}{c} 4 \\ -2 \end{array} \right)
\]

(ii) (a) **Required To Find:** The position vector of \( F \).

**Solution:**
If \( RF = FT \), then \( F \) is the midpoint of \( \overrightarrow{RT} \).
\[
\overrightarrow{OF} = \left( \begin{array}{c} \frac{3+5}{2} \\ \frac{4-2}{2} \end{array} \right) \\
= \left( \begin{array}{c} 4 \\ 1 \end{array} \right)
\]

OR
(b) **Required To State:** Coordinates of \( F \).

**Solution:**

If \( \overrightarrow{OF} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \), then \( F = (4, 1) \).

14. a. **Data:** \( A = (2, 1) \), \( B = \begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix} \) and \( C = (5, 6) \)

\[ AB = C \, , \]

**Required To Calculate:** \( x \) and \( y \).

**Calculation:**

\[ AB = (2, 1) \begin{pmatrix} 1 & x \\ y & -2 \end{pmatrix} \]

\[ = ((2 \times 1) + (1 \times y) \quad (2 \times x) + (1 \times -2)) \]

\[ = (2 + y \quad 2x - 2) \]

If \( AB = C \) then

\[ (2 + y \quad 2x - 2) = (5 \quad 6) \]

Equating corresponding entries.

\[ 2 + y = 5 \]

\[ y = 3 \]

\[ 2x - 2 = 6 \]

\[ 2x = 8 \]

\[ x = 4 \]

\[ \therefore x = 4 \text{ and } y = 3 \]
b. **Data:** \( R = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \).

(i) **Required To Show:** \( R \) is a non-singular matrix.

**Proof:**
\[
\det R = (2 \times 3) - (-1 \times 1) = 6 + 1 = 7
\]
Since \( R \neq 0 \), then \( R \) is non-singular.

(ii) **Required To Find:** \( R^{-1} \)

**Solution:**
\[
R^{-1} = \frac{1}{7} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3/7 & 1/7 \\ -1/7 & 2/7 \end{pmatrix} = \begin{pmatrix} 3/7 & 1 \\ -1/7 & 2 \\ -1/7 & 1/7 \end{pmatrix}
\]

(iii) **Required To Prove:** \( RR^{-1} = I \)

**Proof:**
\[
R \times R^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}
\]
\[
e_{11} = (2 \times 3) + (-1 \times 1) = 1
\]
\[
e_{12} = (2 \times 1) + (-1 \times 2) = 0
\]
\[
e_{21} = (1 \times 3) + (3 \times -1) = 0
\]
\[
e_{22} = (1 \times 1) + (3 \times 2) = 1
\]
\[
\therefore RR^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I
\]

Q.E.D.
(iv) **Data:** $2x - y = 0$ and $x + 3y = 7$

**Required To Calculate:** $x$ and $y$ by matrix method.

**Calculation:**

$2x - y = 0 \ldots (1)$

$x + 3y = 7 \ldots (2)$

\[
\begin{bmatrix}
2 & -1 \\
1 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
0 \\
7
\end{bmatrix}
\]

\[
R \begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 0 \\
7
\end{bmatrix}
\]

$\times R^{-1}$

$R \times R^{-1} \begin{bmatrix}
x \\
y
\end{bmatrix} = R^{-1} \begin{bmatrix}
0 \\
7
\end{bmatrix}$

$I \times \begin{bmatrix}
x \\
y
\end{bmatrix} = R^{-1} \begin{bmatrix}
0 \\
7
\end{bmatrix}$

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix}
\frac{3}{7} & \frac{1}{7} \\
-\frac{1}{7} & \frac{2}{7}
\end{bmatrix}
\begin{bmatrix}
0 \\
7
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{3 \times 0}{7} + \frac{1 \times 7}{7} \\
-\frac{1 \times 0}{7} + \frac{2 \times 7}{7}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 \\
2
\end{bmatrix}
\]

Equating corresponding entries.

$x = 1$ and $y = 2$