JANUARY 2009 CXC MATHEMATICS GENERAL PROFICIENCY (PAPER 2)

Section I

1. a. Required To Calculate: \( \frac{3\frac{3}{4}}{2\frac{1\frac{5}{6}}{3}} \)

Calculation:
Denominator:
\[
2 \frac{1\frac{5}{6}}{3} = \frac{2 \cdot 3 + 5}{6} = \frac{11}{6}
\]

and
\[
3 \frac{3}{4} = \frac{3 \cdot 4 + 3}{4} = \frac{15}{4}
\]

\[
1 \frac{1}{2} = \frac{1 \cdot 2 + 1}{2} = \frac{3}{2}
\]

\[
\frac{15}{4} \times \frac{2}{3} = \frac{5}{2}
\]

\[
= 2 \frac{1}{2}
\]

b. (i) Required To Calculate: Value of one BDS$ in EC$

Calculation:
\[
BDS$2\ 000 = EC$2\ 700
\]

\[
BDS$1 = \frac{2\ 700}{2\ 000} = EC$1.35
\]
(ii) **Required To Calculate:** Amount of BDS$ received for exchanging ECS$432.00  
**Calculation:**  
ECS$1.35 = BDS$1.00  
ECS$1.00 = BDS$\frac{1.00}{1.35}$  
ECS$432 = BDS$\frac{1.00}{1.35} \times 432$  
= BDS$320$

c. **Data:** Principal $24,000 receives 8% per annum compound interest.  
**Required To Calculate:** Amount of money after 2 years.  
**Calculation:**  
Interest at the end of 1\text{st} year = \frac{8}{100} \times 24,000  
= \$1,920$

Principal at start of second year = $24,000 + \$1,920  
= $25,920$

Interest at the end of 2\text{nd} year = \frac{8}{100} \times 25,920  
= \$2,073.60$

Hence, total amount of money in the account at the end of 2 years  
= $25,920 + \$2,073.60  
= \$27,993.60$

**OR**

\[ A = P \left(1 + \frac{R}{100}\right)^n \]

\[ A = 24,000 \left(1 + \frac{8}{100}\right)^2 \]

\[ = 24,000(1.08)^2 \]

\[ = \$27,993.60 \]
2. a. Required To Simplify: \( \frac{2m - 5m}{n - 3n} \)

Solution:
\[
\frac{2m - 5m}{n - 3n} = \frac{3(2m) - 5m}{3n} = \frac{6m - 5m}{3n} = \frac{m}{3n}
\]
as a single fraction

b. Data: \( a \times b = a^2 - b \)

Required To Calculate: \( 5 \times 2 \)

Calculation:
\[
5 \times 2 = (5)^2 - 2 = 25 - 2 = 23
\]

(c. Required To Factorise: \( 3x - 6y + x^2 - 2xy \)

Solution:
\[
3x - 6y + x^2 - 2xy = 3(x - 2y) + x(x - 2y)
= (x - 2y)(3 + x)
\]

d. Data: A 21 cm drinking straw cut into 3 pieces of varying length.

(i) Required To Find: Length of each piece in terms of \( x \).

Solution:
Length of 1\(^{st}\) piece = \( x \) cm (data)
Length of 2\(^{nd}\) piece = \( x - 3 \) cm (data)
Length of 3\(^{rd}\) piece = \( 2 \times x \)
= \( 2x \) cm (data)

(ii) Required To Find: Expression in terms of \( x \) to represent the sum of the lengths of all three pieces.

Solution:
Sum of the lengths of all three pieces of straw
\[
= x + (x - 3) + 2x
= x + x + 2x - 3
= (4x - 3) \text{ cm}
\]
(iii) **Required To Calculate:** \( x \)

**Calculation:**

\[
4x - 3 = 21
\]

\[
4x = 21 + 3
\]

\[
4x = 24
\]

\[
x = \frac{24}{4}
\]

\[
= 6
\]

3. a. **Data:** School of 90 form 5 students where 54 study P.E., 42 Music, \( x \) both and 6 neither.

(i)-(ii) **Required To Complete:** Venn diagram to represent the information given.

**Solution:**

\[
\begin{aligned}
&U \\
&P \\
&M \\
&(54 - x) \\
&x \\
&(42 - x) \\
&6
\end{aligned}
\]

(iii) **Required To Calculate:** \( x \)

**Calculation:**

\[
(54 - x) + x + (42 - x) = 90
\]

\[
102 - x = 90
\]

\[
x = 102 - 90
\]

\[
= 12
\]

b. **Data:** Diagram as shown below.
(i) **Required To Calculate:** Length of $MK$.

**Calculation:**

\[ \frac{MK}{10} = \sin 30^\circ \]

\[ MK = 10 \sin 30^\circ \]

\[ = 5 \text{ m} \]

(ii) **Required To Calculate:** Length of $JK$.

**Calculation:**

\[ JM = \sqrt{(13)^2 - (5)^2} \] \hspace{1cm} (Pythagoras' Theorem)

\[ = \sqrt{144} \]

\[ = 12 \text{ m} \]

Length of $JK = Length$ of $JM - Length$ of $KM$

\[ = 12 - 5 \]

\[ = 7 \text{ m} \]

4. **Data:** Diagram with straight line cutting the axes at $P$ and $Q$.

![Diagram](image)

a. **Required To State:** Coordinates of $P$ and $Q$.

**Solution:**

Line cuts the $y$ – axis at 3, \hspace{0.5cm} \therefore P = (0, 3).

Line cuts the $x$ – axis at – 2, \hspace{0.5cm} \therefore Q = (–2, 0).
b. (i) **Required To Find:** Gradient of \( PQ \).

**Solution:**

Gradient of \( PQ = \frac{3 - 0}{0 - (-2)} \)

\[ = \frac{3}{2} \]

(ii) **Required To Find:** Equation of \( PQ \).

**Solution:**

Equation of \( PQ \) is

\[ \frac{y - 3}{x - 0} = \frac{3}{2} \] Using point at \( P \)

\[ 2y - 6 = 3x \]

\[ 2y = 3x + 6 \]

\[ y = \frac{3}{2} x + 3 \]

**OR**

\[ \frac{y - 0}{x - (-2)} = \frac{3}{2} \] Using point at \( Q \)

\[ 2y = 3x + 6 \]

**OR**

\( y = mx + c \) is the general equation of a straight line where gradient, \( m = \frac{3}{2} \)

and intercept on the vertical axis is 3.

\[ \therefore y = \frac{3}{2} x + 3 \]

c. **Data:** Point \((-8, t)\) lies on \( PQ \).

**Required To Calculate:** \( t \)

**Calculation:**

Equation of \( PQ \) is \[ y = \frac{3}{2} x + 3 \].

\[ t = \frac{3}{2} (-8) + 3 \]

\[ t = -12 + 3 \]

\[ t = -9 \]
(d) **Data:** \( AB \) is perpendicular to \( PQ \) and passes through \((6, 2)\).

**Required To Find:** Equation of \( AB \).

**Solution:**

Gradient of \( PQ = \frac{3}{2} \), then gradient of \( AB = -\frac{2}{3} \).

(Product of the gradients of perpendicular lines = -1).

Equation of \( AB \) is

\[
\frac{y - 2}{x - 6} = -\frac{2}{3}
\]

\[3(y - 2) = -2(x - 6)\]

\[3y - 6 = -2x + 2\]

\[3y = -2x + 12\]

\[3y = -2x + 18\]

\[\div 3\]

\[y = -\frac{2}{3}x + 6\]

is of the form \( y = mx + c \), where \( m = -\frac{2}{3} \) and \( c = 6 \).

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5. **Data:**

![Diagram of a cereal box](image)
a. **Required To Calculate:** Volume of large cereal box.
   **Calculation:**
   \[ \text{Volume} = (25 \times 8 \times 36) \]
   \[ = 7200 \text{ cm}^3 \]

b. **Required To Calculate:** Total surface area of large cereal box.
   **Calculation:**
   Area of front and back faces \[= 2(25 \times 36) \text{ cm}^2 \]
   Area of left and right faces \[= 2(8 \times 36) \text{ cm}^2 \]
   Area of base and top faces \[= 2(25 \times 8) \text{ cm}^2 \]

   Total area of large cereal box
   \[= 2(25 \times 36) + 2(8 \times 36) + 2(25 \times 8) \]
   \[= 1800 + 576 + 400 \]
   \[= 2776 \text{ cm}^2 \]

c. **Data:** One large box can fill six small boxes each of equal volume.
   (i) **Required To Calculate:** Volume of one small cereal box.
       **Calculation:**
       Volume of large cereal box \[= 6 \times \text{Volume of small cereal box} \]
       Volume of small cereal box \[= \frac{7200}{6} \]
       \[= 1200 \text{ cm}^3 \]

   (ii) **Required To List:** Two different pairs of values which the company can use for the height and width of a small box.
       Solution:

       In a small cereal box, let
       length \(= l \) cm and width \(= w \) cm.
There are an infinite number of possible choices for values of $l$ and $w$ such that $l \times w = 60$.
For simplicity, we may choose two integral values, such as
Let $l = 10$ and $w = 6$ and $l = 12$ and $w = 5$ as the two pairs of values.

6. a. **Required To Construct:** Rectangle $PQRS$ with $PQ = 7.0$ cm and $QR = 5.5$ cm.
**Solution:**

![Rectangle PQRS Diagram]

b. **Data:** $\Delta L \rightarrow \Delta M$
(i) **Required To Calculate:** Translation vector that maps $\Delta L$ onto $\Delta M$.
**Calculation:**

![Translation Vector Diagram]
L is mapped onto M by a horizontal shift of 4 units to the right and 4 units vertically upwards. This may be represented by the translation vector, \( T \), where
\[ T = \begin{pmatrix} 4 \\ 4 \end{pmatrix}. \]

(ii) **Data:** \( \triangle L \rightarrow \triangle N \), by an enlargement, say \( E \).

(a) **Required To Find:** Centre of enlargement, \( G \) on diagram.

**Solution:**

(b) **Required To State:** Coordinates of \( G \).

**Solution:**

\( G \) is (-5, 0), as seen on the diagram.

(c) **Required To Calculate:** Scale factor of the enlargement.

**Calculation:**
Measuring the image length and its corresponding object length.

\[
\frac{\text{Image Length}}{\text{Object Length}} = \frac{6}{3} = 2
\]

Therefore, scale factor of the enlargement is 2.

7. **Data**: Given table of values for marks in a test obtained by 70 students.
   a. **Required To Complete**: Table to show cumulative frequency distribution.
      **Solution**:
      Modifying the table.
      Distribution – discrete variable.

<table>
<thead>
<tr>
<th>Marks</th>
<th>Frequency (f)</th>
<th>Cumulative frequency (C.F.)</th>
<th>Points to be plotted (U.C.B, C.F.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.C.B U.C.B</td>
<td>(f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 – 10</td>
<td>2</td>
<td>2</td>
<td>(10, 2)</td>
</tr>
<tr>
<td>11 – 20</td>
<td>5</td>
<td>7</td>
<td>(20, 7)</td>
</tr>
<tr>
<td>21 – 30</td>
<td>9</td>
<td>16</td>
<td>(30, 16)</td>
</tr>
<tr>
<td>31 – 40</td>
<td>14</td>
<td>14 + 16 = 30</td>
<td>(40, 30)</td>
</tr>
<tr>
<td>41 – 50</td>
<td>16</td>
<td>16 + 30 = 46</td>
<td>(50, 46)</td>
</tr>
<tr>
<td>51 – 60</td>
<td>12</td>
<td>12 + 46 = 58</td>
<td>(60, 58)</td>
</tr>
<tr>
<td>61 – 70</td>
<td>8</td>
<td>8 + 58 = 66</td>
<td>(70, 66)</td>
</tr>
<tr>
<td>71 – 80</td>
<td>4</td>
<td>4 + 66 = 70</td>
<td>(80, 70)</td>
</tr>
</tbody>
</table>

A cumulative frequency curve must start from the horizontal axis. By checking the values of U.C.B., we find that an initial point with U.C.B = 0 and C.F. = 0.

b. (i) **Required To Draw**: Cumulative frequency curve for the information given.
(ii) **Required To State:** Assumption made when drawing the curve through the point \((0, 0)\).

**Solution:**
In drawing the curve with a starting point at \((0, 0)\), we are assuming that no student obtained a mark of 0. This also seems clear from the table of values that is given.

c. **Required To Find:** Number of students who passed the test.

**Solution:**
If the pass mark is 47 (which corresponds to a cumulative frequency value of 41), then the number of students who passed the test is \(70 - 41 = 29\).

d. **Required To Calculate:** Probability randomly chosen student had a mark less than or equal to 30.
Calculation:

\[ P(\text{Student obtained a mark} \leq 30) = \frac{\text{No. of students who obtained a mark} \leq 30}{\text{Total no. of students}} = \frac{16}{70} = \frac{8}{35} \]

8. **Data:** Pattern made up of lines and dots.

   a. **Required To Draw:** Fourth diagram in the sequence.

   **Solution:**
   The fourth diagram in the sequence is
   ![Diagram](image)

   b. **Required To Complete:** Table given.

   **Solution:**

<table>
<thead>
<tr>
<th>No. of dots, (d)</th>
<th>Pattern connecting (l) and (d)</th>
<th>No. of line segments, (l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 (= 2 \times 5 = -4)</td>
<td>6 (= 2 - 4)</td>
<td>12</td>
</tr>
<tr>
<td>8 (= 2 \times 8 = -4)</td>
<td>18 (= 2 - 4)</td>
<td>12</td>
</tr>
<tr>
<td>11 (= 2 \times 11 = -4)</td>
<td>18 (= 2 - 4)</td>
<td>12</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>(i) 62 (= 2 \times 62 = -4)</td>
<td>120 (= 2 - 4)</td>
<td>12</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>(ii) 92 (= 2 \times 92 = -4)</td>
<td>180 (= 2 - 4)</td>
<td></td>
</tr>
</tbody>
</table>

   c. (i) **Required To Find:** No. of dots in the 6\(^{th}\) diagram of the sequence.

   **Solution:**
   The sequence of dots \(d\) is

   \[ d = 5 \quad 8 \quad 11 \quad 14 \quad 17 \]

   \[ 5 + 3 = 8 \quad 2^{nd} \]

   \[ 8 + 3 = 11 \quad 3^{rd} \]

   \[ 11 + 3 = 14 \quad 4^{th} \]

   \[ 14 + 3 = 17 \quad 5^{th} \]

   \[ 17 + 3 = 20 \quad 6^{th} \]

   \[ \therefore \text{In the 6}^{th}\text{ diagram of the sequence, the number of dots, } d = 20. \]
(ii) **Required To Find:** Number of line segments in the 7th diagram of the sequence.

**Solution:**
In the 7th diagram
\[ d = 20 \]
6th
\[ 3 + 20 = 23 \]
7th

\[ \therefore \text{No. of line segments} = 2 \times 23 - 4 \]
\[ = 46 - 4 \]
\[ = 42 \]

(iii) **Required To Find:** The rule which relates \( l \) to \( d \).

**Solution:**
By observing the pattern values of \( d \) and \( l \)

<table>
<thead>
<tr>
<th>( d )</th>
<th>( l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( 2 \times 5 - 4 )</td>
</tr>
<tr>
<td>8</td>
<td>( 2 \times 8 - 4 )</td>
</tr>
<tr>
<td>11</td>
<td>( 2 \times 11 - 4 )</td>
</tr>
<tr>
<td>62</td>
<td>( 2 \times 62 - 4 )</td>
</tr>
<tr>
<td>92</td>
<td>( 2 \times 92 - 4 )</td>
</tr>
</tbody>
</table>

\[ l = 2 \times d - 4 \]
\[ l = 2d - 4 \]

9. a. **Data:** \[ \frac{p}{2} = \sqrt{\frac{t + r}{g}} \]

**Required To Make:** \( t \) the subject of the formula.

**Solution:**
\[ \frac{p}{2} = \sqrt{\frac{t + r}{g}} \]

Squaring to remove \( \sqrt{ } \)
\[ \left( \frac{p}{2} \right)^2 = \left( \sqrt{\frac{t + r}{g}} \right)^2 \]
\[ \frac{p^2}{4} = \frac{t + r}{g} \]
\[ g \times p^2 = 4(t + r) \]
\[ gp^2 = 4t + 4r \]
\[ gp^2 - 4r = 4t \]
\[ 4t = gp^2 - 4r \]
\[ t = \frac{gp^2 - 4r}{4} \text{ or } \frac{gp^2}{4} - r \]

b. **Data:** \( f(x) = 2x^2 - 4x - 13 \)

(ii) **Required To Express:** \( f(x) \) in the form \( f(x) = a(x + h)^2 + k \).

**Solution:**

\( f(x) = 2x^2 - 4x - 13 \)
\[ = 2(x^2 - 2x) - 13 \]

Half the coefficient of \(-2x = \frac{1}{2}(-2)\)
\[ = -1 \]

\[ f(x) = 2(x - 1)^2 + * \]
\[ 2(x^2 - 2x + 1) = 2x^2 - 4x + 2 \]

\[ -15 = * \]
\[ -13 * \]

Hence, \( f(x) = 2x^2 - 4x - 13 \equiv 2(x - 1)^2 - 15 \) is of the form \( a(x + h)^2 + k \), where \( a = 2, h = -1 \) and \( k = -15 \).

**OR**

\[ 2x^2 - 4x - 13 = a(x + h)^2 + k \]
\[ = a(x^2 + 2hx + h^2) + k \]
\[ = ax^2 + 2ahx + ah^2 + k \]

Equating coefficient of \( x^2 \).
\[ a = 2 \]

Equating coefficient of \( x \).
\[ -4 = 2(2)h \]
\[ h = -1 \]

Equating constants.
\[ 2(-1)^2 + k = -13 \]
\[ k = -15 \]

\[ \therefore 2x^2 - 4x - 13 \equiv 2(x - 1)^2 - 15 \], where \( a, h \) and \( k \) are already given.
(ii) **Required To Find:** Values of \( x \) at which \( f(x) \) cuts the \( x \)– axis  

**Solution:**  
Assuming the graph is \( f(x) = 2x^2 - 4x - 13 \), \( f(x) \) cuts the \( x \)– axis at \( f(x) = 0 \).  
Let  
\[
2x^2 - 4x - 13 = 0
\]
\[
x = \frac{(-4) \pm \sqrt{(-4)^2 - 4(2)(-13)}}{2(2)}
\]
\[
= \frac{4 \pm \sqrt{16 + 104}}{4}
\]
\[
= \frac{4 \pm \sqrt{120}}{4}
\]
\[
= \frac{4 \pm 4\sqrt{30}}{4}
\]
\[
= 1 \pm \frac{\sqrt{30}}{2}
\]
\[
\therefore f(x) \text{ cuts the } x \text{– axis at } x = 1 + \frac{\sqrt{30}}{2} \text{ and } 1 - \frac{\sqrt{30}}{2}.
\]

(iii) **Required To Find:** Interval for which \( f(x) \leq 0 \).  

**Solution:**  
\( f(x) = 2x^2 - 4x - 13 \). Coefficient of \( x^2 > 0 \), therefore \( f(x) \) is a parabola in shape with a minimum point.

\[
f(x) \leq 0 \text{ for } \left\{ x : 1 - \frac{\sqrt{30}}{2} \leq x \leq 1 + \frac{\sqrt{30}}{2} \right\}
\]
(iv) **Required To Find:** Minimum value of \( f(x) \).

**Solution:**

\[
\begin{align*}
    f(x) &= 2x^2 - 4x - 13 \\
    &= 2(x - 1)^2 - 15 \\
    2(x - 1)^2 &\geq 0 \quad \forall x \\
    \therefore f(x)_{\text{min}} &= 0 - 15 \\
    &= -15
\end{align*}
\]

**OR**

\[
\begin{align*}
    f(x) &= 2x^2 - 4x - 13 \\
    \text{The axis of symmetry of } f(x) \text{ is } x &= \frac{-(-4)}{2(2)} \\
    &= 1 \\
    \text{The axis of symmetry passes through the minimum point and hence } x &= 1 \text{ at the minimum point.} \\
    f(1) &= 2(1)^2 - 4(1) - 13 \\
    &= -15 \\
    \therefore f(x)_{\text{min}} &= -15
\end{align*}
\]

**OR**

The \( x \) coordinates of the minimum point is half way between the \( x \) value and which \( f(x) \) cuts the \( x \)-axis is

\[
\begin{align*}
    x_{\text{min}} &= \frac{1 + \sqrt{30}}{2} + \frac{1 - \sqrt{30}}{2} \\
    &= \frac{2}{2} \\
    &= 1 \\
    f(1) &= -15 \\
    \therefore f(x)_{\text{min}} &= -15
\end{align*}
\]
Required To Find: $x$ at which $f(x)$ is minimum.

Solution:

$$f(x) = 2(x - 1)^2 - 15$$

$$f(x)_{\text{min}} = -15 \text{ at } 2(x - 1)^2 = 0$$

That is, $(x - 1)^2 = 0$ and $x = 1$.

10. a. Data: $f : x \rightarrow x - 3$ and $g : x \rightarrow x^2 - 1$.

(i) Required To Calculate: $f(6)$.

Calculation:

$$f(6) = 6 - 3$$

$$= 3$$

(ii) Required To Calculate: $f^{-1}(x)$.

Calculation:

Let

$$y = x - 3$$

$$y + 3 = x$$

$$x = y + 3$$

Replace $y$ by $x$

$$f^{-1} : x \rightarrow x + 3$$

(iii) Required To Prove: $fg(2) = fg(-2) = 0$.

Proof:

$$g(2) = (2)^2 - 1$$

$$= 4 - 1$$

$$= 3$$

$$fg(2) = f(3)$$

$$= 3 - 3$$

$$= 0$$

$$g(-2) = (-2)^2 - 1$$

$$= 4 - 1$$

$$= 3$$

$$fg(-2) = f(3)$$

$$= 3 - 3$$

$$= 0$$

$$fg(2) = fg(-2) = 0$$

Q.E.D.
b. **Data:** A distance time graph for a train travelling between stations A and B.

(i) **Required To Find:** Time during with train was a rest.

**Solution:**

![Distance Time Graph](attachment:image)

Train was at rest at B for $60 - 40 = 20$ minutes.

(ii) **Required To Find:** Average speed of the train from A to B.

**Solution:**

![Distance Time Graph](attachment:image)

Average speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

$= \frac{100 \text{ km}}{(40 - 0) \text{ hr}}$

$= \frac{100}{2} \text{ kmh}^{-1}$

$= 150 \text{ kmh}^{-1}$

(iii) **Required To Find:** Time taken for the train to travel from B to C.

**Solution:**

Time taken from B to C $= \frac{\text{Distance from B to C}}{\text{Average speed from B to C}}$

$= \frac{50 \text{ km}}{60 \text{ kmh}^{-1}}$

$= \frac{5}{6} \text{ hour}$

$= 50 \text{ minutes}$
(iv) **Required To Draw:** Line segment which describes the journey from B to C.
**Solution:**

![Graph of y = \( \frac{1}{2} \tan x \)](image)

11. a. **Data:** Table of values for \( y = \frac{1}{2} \tan x \).

(i) **Required To Complete:** Table of values given.
**Solution:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.09</td>
<td>0.18</td>
<td>0.29</td>
<td>0.42</td>
<td>0.60</td>
<td>0.87</td>
</tr>
</tbody>
</table>

**Note:** When \( x = 10° \), \( y = 0.09 \) which was erroneously given as 0.13 in the exam paper. This value has been altered.

(ii) **Required To Draw:** Graph of \( y = \frac{1}{2} \tan x \).
**Solution:**

![Graph of y = \( \frac{1}{2} \tan x \)](image)
(iii) **Required To Find:** \( x \) when \( y = 0.7 \).

**Solution:**
When \( y = 0.7 \), \( x = 54.5^\circ \) (Read off).

b. **Data:** Diagram as shown below.

![Diagram](image)

(i) **Required To Calculate:** \( \hat{OAE} \)

**Calculation:**
\( \hat{OAE} = 90^\circ \)
(The angle made by a tangent to a circle and a radius, at the point of contact = 90°). Similarly for \( \hat{OBE} \).

(ii) **Required To Calculate:** \( \hat{AOB} \)

**Calculation:**
\[
\hat{AOB} = 360^\circ - (90^\circ + 90^\circ + 48^\circ) \\
= 132^\circ
\]
(The sum of the angles in a quadrilateral = 360°).

(iii) **Required To Calculate: \( \hat{A}\hat{C}\hat{B} \)**
    **Calculation:**
    A chord \( AB \) subtends twice the angle at the centre of a circle, that it
    subtends at the circumference, standing on the same arc.
    \[
    \therefore \hat{A}\hat{C}\hat{B} = \frac{1}{2}(132°) = 66°
    \]

(iv) **Required To Calculate: \( \hat{A}\hat{D}\hat{B} \)**
    **Calculation:**
    \[
    \hat{A}\hat{D}\hat{B} = 180° - 66° = 114°
    \]
    (The opposite angles of cyclic quadrilateral are supplementary).

12. a. **Data:** Diagram as shown below.

![Diagram](image)

(i) **Required To Calculate: Length of \( UW \).**
    **Calculation:**
    \[
    UW^2 = (8)^2 + (10)^2 - 2(8)(10)\cos 60° \quad \text{(Cosine Rule)}
    \]
    \[
    = 64 + 100 - 160 \times \frac{1}{2}
    \]
    \[
    = 164 - 80
    \]
    \[
    = 84
    \]
    \[
    UW = \sqrt{84} = 9.165 \quad \text{(to 2 decimal places)}
    \]
(ii) Required To Calculate: $U\hat{V}W$
Calculation:
\[
\frac{9.165}{\sin U\hat{V}W} = \frac{11}{\sin 40^\circ} \\
\sin U\hat{V}W = \frac{9.165 \sin 40^\circ}{11} \\
= 0.535 \\
U\hat{V}W = \sin^{-1}(0.535) \\
= 32.34^\circ \\
= 32.3^\circ \text{ (to the nearest } 0.1^\circ) \\
\]

(iii) Required To Calculate: Area of $\triangle TUW$
Calculation:
\[
\text{Area} = \frac{1}{2} (8 \times 10) \sin 60^\circ \\
= 34.64 \text{ cm}^2 \\
\]

b. NO SOLUTION HAS BEEN OFFERED AS THIS PART OF THE QUESTION IS ON EARTH GEOMETRY WHICH HAS BEEN REMOVED FROM THE COURSE.

13. Data: Diagram showing vectors $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.

\[
\text{a. (i) Required To Express: } \overrightarrow{OP} \text{ in the form } \begin{pmatrix} x \\ y \end{pmatrix}. \\
\text{Solution:} \\
P \text{ is the point (3, 2), as seen on the diagram.} \\
\]

\[ \overrightarrow{OP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \] is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = 3 \) and \( y = 2 \).

(ii) **Required To Express:** \( \overrightarrow{OQ} \) in the form \( \begin{pmatrix} x \\ y \end{pmatrix} \).

**Solution:**

\( Q \) is the point (-1, 3) as seen on the diagram.

\[ \overrightarrow{OQ} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \] is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = -1 \) and \( y = 3 \).

b. **Data:** \( R \) has coordinates (8, 9).

(i) **Required To Express:** \( \overrightarrow{OR} \) in the form \( \begin{pmatrix} x \\ y \end{pmatrix} \).

**Solution:**

\( R \) is (8, 9).

\[ \overrightarrow{OR} = \begin{pmatrix} 8 \\ 9 \end{pmatrix} \]

\[ \overrightarrow{QR} = \overrightarrow{OQ} + \overrightarrow{OR} \]

\[ = \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 8 \\ 9 \end{pmatrix} \]

\[ = \begin{pmatrix} 7 \\ 12 \end{pmatrix} \]

is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = 7 \) and \( y = 12 \).

(ii) **Required To Prove:** \( \overrightarrow{OP} \) is parallel to \( \overrightarrow{QR} \).

**Proof:**

\[ \frac{x}{3} = \frac{y}{2} \]

\[ \frac{x}{9} = \frac{y}{6} \]

is of the form \( \begin{pmatrix} x \\ y \end{pmatrix} \) where \( x = 9 \) and \( y = 6 \).
\[
\overrightarrow{OP} = \left(\frac{3}{2}\right)
\]
\[
\overrightarrow{QR} = \left(\frac{9}{6}\right)
\]
\[
= 3 \left(\frac{3}{2}\right)
\]
\[
\overrightarrow{QR} \text{ is a scalar multiple (3) of } \overrightarrow{OP}, \text{ hence } \overrightarrow{OP} \text{ and } \overrightarrow{QR} \text{ are parallel.}
\]

(iii) **Required To Find:** \( |\overrightarrow{PR}| \).

**Solution:**
\[
\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR}
\]
\[
= \left(-\frac{3}{2}\right) + \left(\frac{8}{9}\right)
\]
\[
= \left(\frac{5}{7}\right)
\]

Magnitude of \( \overrightarrow{PR} = |\overrightarrow{PR}| \)
\[
= \sqrt{(5)^2 + (7)^2}
\]
\[
= \sqrt{25 + 49}
\]
\[
= \sqrt{74} \text{ units}
\]
c. Data: \( S = (a, b) \).

(i) Required To Find: \( \overrightarrow{QS} \) in terms of \( a \) and \( b \).

Solution:

\[
\overrightarrow{OS} = \begin{pmatrix} a \\ b \end{pmatrix}
\]

\[
\overrightarrow{QS} = \overrightarrow{OO} + \overrightarrow{OS}
\]

\[
= \begin{pmatrix} -1 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}
\]

\[
= \begin{pmatrix} a + 1 \\ b + 3 \end{pmatrix}
\]

in terms of \( a \) and \( b \).

(ii) Data: \( \overrightarrow{QS} = \overrightarrow{OP} \)

Required To Calculate: \( a \) and \( b \).

Calculation:
If

\[
\begin{pmatrix} a + 1 \\ b + 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}
\]

Equating components.

\( a + 1 = 3 \)

\( a = 2 \)

and

\( b + 3 = 2 \)

\( b = 5 \)
(iii) **Required To Prove:** \( OPSQ \) is a parallelogram.

**Proof:**

If \( QS = OP \) then \( |QS| = |OP| \) and \( QS \) is parallel to \( OP \). If one pair of opposite sides of a quadrilateral is both parallel and equal, then the quadrilateral is a parallelogram. Therefore, \( OPSQ \) is a parallelogram.

14. a. **Data:** \[ A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \] and \[ B = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}. \]

**Required To Calculate:** \( 3AB \).

**Calculation:**
\[ A_{2 \times 2} \times B_{2 \times 2} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \]

\[ e_{11} = (1 \times 1) + (2 \times 2) = 5 \]

\[ e_{12} = (1 \times 3) + (2 \times 5) = 13 \]

\[ e_{21} = (2 \times 1) + (1 \times 2) = 4 \]

\[ e_{22} = (2 \times 3) + (1 \times 5) = 11 \]

\[ AB = \begin{pmatrix} 5 & 13 \\ 4 & 11 \end{pmatrix} \]

\[ 3AB = 3 \begin{pmatrix} 5 & 13 \\ 4 & 11 \end{pmatrix} = \begin{pmatrix} 15 & 39 \\ 12 & 33 \end{pmatrix} \]

b. Data:

\[ \begin{pmatrix} 5 & 13 \\ 4 & 11 \end{pmatrix} \]

\[ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \]

\[ \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \]

(i) **Required To State:** Effect of \(V\) on \(\triangle ABC\).

**Solution:**

\[ \triangle ABC \rightarrow \triangle ABC' \]

The image of \(\triangle ABC\) under \(V\) is an enlargement, centre \(O\) and scale factor 2.

(ii) **Required To Find:** \(2 \times 2\) matrix that represents the combined transformation of \(V\) followed by \(W\).
Solution:
The combined transformation \( V \) followed by \( W \) is expressed as \( WV \).
\[
WV = \begin{pmatrix}
-1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
2 & 0 \\
0 & 2
\end{pmatrix}
= \begin{pmatrix}
-1 \times 2 + (0 \times 0) & -1 \times 0 + (0 \times 2) \\
(0 \times 2) + (1 \times 0) & (0 \times 0) + (1 \times 2)
\end{pmatrix}
= \begin{pmatrix}
-2 & 0 \\
0 & 2
\end{pmatrix}
\]

(iii) **Required To Find:** Coordinates of the image of \( \Delta ABC \) under the combined transformation.

**Solution:**
The image of \( \Delta ABC \) under \( WV \)
\[
\begin{pmatrix}
-2 & 0 \\
0 & 2
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 2 \\
2 & 1 & 1
\end{pmatrix}
= \begin{pmatrix}
-2 & -2 & -4 \\
4 & 2 & 2
\end{pmatrix}
\]

\( A \) is mapped onto \((-2, 4)\).
\( B \) is mapped onto \((-2, 2)\).
\( C \) is mapped onto \((-4, 2)\).

c. **Data:** \( 11x + 6y = 6 \) and \( 9x + 5y = 7 \).

(i) **Required To Express:** The two equations in the form \( AX = B \).

**Solution:**
\( 11x + 6y = 6 \)
\( 9x + 5y = 7 \)

may be expressed as
\[
\begin{pmatrix}
11 & 6 \\
9 & 5
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix}
= \begin{pmatrix}
6 \\
7
\end{pmatrix}
\]
is of the form \( AX = B \), where
\[
A = \begin{pmatrix}
11 & 6 \\
9 & 5
\end{pmatrix},
X = \begin{pmatrix}
x \\
y
\end{pmatrix}
\text{ and } B = \begin{pmatrix}
6 \\
7
\end{pmatrix}.
\]

(ii) **Required To Calculate:** \( x \) and \( y \).

**Calculation:**
\[ A = \begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix} \]

\[ \text{det } A = (11 \times 5) - (6 \times 9) \]
\[ = 55 - 54 \]
\[ = 1 \]

\[ A^{-1} = \frac{1}{1} \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix} \]
\[ = \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix} \]

\[ AX = B \]
\[ \times A^{-1} \]
\[ A \times A^{-1} \times X = A^{-1} \times B \]
\[ I \times X = A^{-1}B \]
\[ \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix} \begin{pmatrix} 6 \\ 7 \end{pmatrix} \]
\[ = \begin{pmatrix} (5 \times 6) + (-6 \times 7) \\ (-9 \times 6) + (11 \times 7) \end{pmatrix} \]
\[ = \begin{pmatrix} -12 \\ 23 \end{pmatrix} \]

Equating corresponding entries.
\[ x = -12 \text{ and } y = 23. \]